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Time-series implications of the permanent income hypothesis on durable goods consumption

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

The permanent income hypothesis has been the standard textbook framework for describing aggregate consumption behavior since it was introduced by Friedman (1957). Hall (1978) formulated the time series representation of the permanent income hypothesis using the rational expectations optimization framework (RE-PIH) and showed that nondurable consumption expenditures follow a random walk process. Mankiw (1982) applied Hall's intertemporal optimization framework to durable goods consumption and derived the result that the change in consumption expenditures on durable goods should follow a first-order moving average process with the MA coefficient equal to negative one plus the depreciation rate of the durable good stock. Using quarterly seasonally adjusted postwar U.S. data, he found, contrary to the theory, that the change in consumer durable expenditures behave as white noise rather than an MA (1). The null hypothesis that the MA coefficient is zero, or the depreciation rate is equal to one, cannot be rejected at conventional significance levels which implies that durable goods are entirely consumed within the period they are purchased. This violates the basic nature of durable goods, which is that they should provide services for more than one period. Thus, the empirical finding that the time series behavior of durable expenditures exhibits the same type of behavior as the nondurable expenditures has been a puzzle.

Caballero (1990) reexamined the durable goods puzzle and showed that the change in consumer durable goods expenditures could follow a higher order MA process if consumers adjust their durable stocks with lags upon an income innovation. He argued that a parsimonious MA (1) process is not likely to detect the spread out consumer responses to the income news and showed that the sum of MA coefficients from estimating annual changes of consumer durable expenditures with a nonparsimonious MA (q) process is significant and approximates the magnitude predicted by the theory. Caballero (1990) concluded that the frictionless RE-PIH model fails to predict the short-run dynamics of durable expenditures but it provides a reasonable explanation of the long run response of durables to aggregate shocks.

In the subsequent paper, Caballero (1993) argued that the delayed adjustment could reflect the infrequent and lumpy microeconomic purchases of consumer durables due to adjustment costs involved in purchasing durable goods. He pointed out that a convex adjustment cost specification, widely used to account for adjustment cost in the representative agent framework, disregards the micro-level observations that consumers purchase durables in lump-sums and infrequently. A problem with a representative agent model with such realistic discontinuous adjustment features is that it cannot be applied directly to aggregate time-series data. The time aggregation problem has to be addressed in this case because different consumers will adjust their durable stocks in different periods. Cabellero (1993) dealt with this aggregation problem by shifting the focus away from the rational expectations optimization framework and focusing on the distributional dynamics of durable good stock. He developed a framework in the context of (S, s) inventory model in which a dynamic analysis of the cross sectional distribution of the durable good stock is made operational. Caballero (1993) showed that the problem of describing the dynamic behavior of aggregate durable good stock can be reduced to that of describing the dynamic behavior of the cross sectional distribution of the deviation variable which is defined as the difference between the actual stock and the optimal target stock determined by the frictionless RE-PIH model. The time series implication of the dynamic framework is that the durable goods expenditure is a sum of the value predicted by the frictionless RE-PIH model and a noise factor associated with nonconvex adjustment. The framework allows one to explain the time-series departure between the actual durable expenditures and the durable expenditures predicted by the frictionless RE-PIH model. He showed that the durable good subcategory subject to larger transaction cost and the sample periods with larger aggregate uncertainty exhibit larger departures from the frictionless RE-PIH model.

Caballero (1993) attempted to describe the aggregate dynamics of intermittent durable good purchases by essentially abandoning the rational expectations optimization model and introducing a framework based on (S, s) inventory rule in which a dynamic aggregation of stochastically heterogeneous units is made operational. Caballero's work contributed by characterizing the connection between the microeconomic intermittent actions and aggregate dynamics at the expense of an empirically more tractable model. The objective of this thesis is to explain the aggregate dynamics of the durable goods expenditures by modeling intermittent durable good purchases within the rational expectation optimization framework. The connection between the discontinuous representative agent model and continuous aggregate data is made by explicitly aggregating the microeconomic outcome across the aggregate data sampling interval. The paper presents a time-series representation of the permanent income model based on a rational expectations optimization framework that is capable of explaining the quarterly aggregate durable goods expenditure series.

In the first part of the analysis, the implicit assumption of the standard RE-PIH model that consumers incur durable expenditures every period is relaxed. The standard stock flow identity is modified to reflect the infrequent purchases of durable goods. The representative agent model incorporating the modification (the base model) predicts that the change in durable expenditures is a function of the durable goods purchase interval. Mankiw's original model is shown to be a special case when the purchase interval is equal to the quarterly data sampling interval. The connection between the discontinuous micro-level model and continuous aggregate data is made by explicitly aggregating the microeconomic outcome across the aggregate data sampling interval.

In the following section, the base model is further generalized by relaxing the assumption that preferences are time separable. The model incorporating the time non-separable preferences in the form of habit persistence is presented. In the subsequent section, the base model is extended to incorporate the seasonal variation observed in the durable expenditure series. The analysis shows how the Seasonal ARIMA model widely used to model seasonality could be derived from the RE-PIH framework proposed in this paper. Seasonally unadjusted data can be directly applied to the stochastic seasonal model, eliminating concerns of any distortions that the seasonal adjustment procedure may have in analyzing the model.

The paper is organized as follows. Chapter 2 goes over the standard RE-PIH model on durable consumption and reviews Caballero's work. The nonconvex adjustment problem, habit persistence effect, and seasonal adjustment issues are discussed in more detail. In chapter 3, a theoretical framework is laid out. First, the frictionless base model is presented. In the subsequent sections, models incorporating time non-separable preference, and seasonal variation are presented. Chapter 4 discusses the data and estimation issues. Empirical results are presented in chapter 5 and conclusions are offered in chapter 6.

2. LITERATURE REVIEW

2.1. The durable goods puzzle

Ever since the "Euler equation approach" pioneered by Hall (1978) dominated the study of consumption, the joint hypothesis of Rational Expectations and the Permanent Income Hypothesis (RE-PIH) has had only limited empirical success. The RE-PIH framework assumes that the agent will utilize the expected future stream of discounted income and current wealth to determine the consumption path. The RE-PIH implies that consumption should follow a random walk; thus, the change in consumption should not be predictable by any other variables. The empirical finding that other variables, especially lagged income changes, can predict consumption changes is referred to as the excess sensitivity puzzle.

Attempts to test the validity of the RE-PIH, however, have been focused mainly on nondurable consumption as opposed to durables. The asymmetric treatment can be attributed to the fact that durables are not entirely consumed during the period in which they are purchased. Since the theory is about the service flow of actual consumption but what is observed in practice is expenditure data, researchers tend to focus away from the durable consumption aspect of aggregate consumption. Despite these difficulties with the data, the study of durable consumption is essential in understanding the business cycle implications because durable expenditures are the most cyclically sensitive element of consumption expenditures.

Mankiw (1982) was the first to examine durable expenditures under the RE-PIH framework. He derived the result, using Hall's optimization framework, that durable expenditures should follow an ARMA (1,1) process as opposed to the AR (1) process for nondurables. The optimization problem is set up as follows.

The representative consumer's optimization problem is to maximize

$$E_t \sum_{s=0}^{T-t} (1+\theta)^{-s} U(K_{t+s}), \quad \text{subject to} \sum_{s=0}^{T-t} (1+r)^{-s}(K_{t+s} - (1-\delta)K_{t+s-1} - Y_{t+s}) = W_t$$

where,

E_t = the mathematical expectation conditional on all information available in time t

θ = rate of subjective time preference

r = real rate of interest, assumed constant over time

U( ) = strictly concave one-period utility function

K_t = stock of durable goods

Y_t = earnings, the only source of uncertainty

W_t = asset apart from human capital

δ = depreciation rate of the consumer's durable stock.

Replacing the consumption flow variable C with the durable stock variable K in the utility function is justified by assuming that the service flows from the durables are proportional to the durable stock. Consumption expenditures in the budget constraint are replaced by the stock adjustment term using the fundamental identity between the stock of durables K and the flow of durables expenditures C,

$$K_t = (1-\delta)K_{t-1} + C_t. \qquad (2.1)$$

Mankiw derived the following Euler equation from solving the optimization problem,

$$E_t U'(K_{t+1}) = [(1+\theta)/(1+r)]\ U'(K_t) \qquad (2.2)$$

According to Mankiw (1982), the Euler equation (2.2) implies that no information available in period t other than K_t helps to predict K_{t+1}. The marginal utility of consumption next period, which is proportional to the durable stock next period, is expected to be same as the discounted marginal utility of consumption this period. This result reflects the desire of a rational forward looking consumer to keep the marginal utility of consumption constant over time.

If the utility function is quadratic, then K_t follows an AR (1) process,

$$K_{t+1} = a_0 + a_1 K_t + \mu_{t+1} \qquad (2.3)$$

where μ_t is serially uncorrelated and $a_1 = (1+\theta)/(1+r)$

Substituting equation (2.3) into equation (2.1), Mankiw derived the ARMA (1,1) closed form equation for durable expenditures C:

$$C_{t+1} = \delta a_0 + a_1 C_t + \mu_{t+1} - (1-\delta)\mu_t \qquad (2.4)$$

With the additional assumption that the rate of time preference equals the real rate of interest ($\theta = r$), Mankiw's ARMA (1,1) model implies that the change in durable expenditures should follow a first-order moving average process, MA (1), with the MA coefficient equal to negative one plus the depreciation rate. Thus with 5% quarterly depreciation, the MA coefficient should approximate - 0.95. Using quarterly U.S. data, he found that, contrary to what is implied by the theory, a null hypothesis that the MA coefficient is zero could not be rejected at conventional significance level. A zero MA coefficient translates into the depreciation rate being equal to one, which is inconsistent with the theory that durable purchases provide services for more than one period. Mankiw interpreted this empirical finding that the aggregate consumer durable expenditures can be well approximated by a random walk process as evidence against the rational expectation-permanent income joint hypothesis. The theory also suggests that in the limiting case where the depreciation rate approaches zero, durable expenditures should follow a white noise process. Thus, the empirical finding that consumer durable expenditures are highly serially correlated, well approximated by a random walk process, has been a puzzle.

The durable goods puzzle remained an unsolved puzzle until Caballero (1990) introduced the "slow adjustment" argument that everybody faces the same income shocks but react to them with different delays. He showed that the change in consumer durable expenditures could follow a higher order MA process if different consumers adjust their durable stocks at different lags upon a wealth innovation. He pointed out that Mankiw's parsimonious MA (1) model is not likely to detect spread out consumer responses and showed that the sum of MA coefficients from estimating annual changes of consumer durable expenditures with a nonparsimonious MA (q) process is significant and approximates the value of –0.95 predicted by the theory. He reinforced this evidence by plotting the sum of the quarterly autocorrelations for changes in durable expenditures and showing that they converge to a number close to -0.5 after longer lags, a value consistent with the 5% quarterly depreciation rate. Hong (1996) applied the MA (q) model to the annual data on durables expenditures of six OECD countries and showed that the sum of the coefficients for the MA(5) process ranged from –0.833 to –0.965. Caballero (1990) concluded that the frictionless RE-PIH model fails to predict the short-run dynamics of durable expenditures but is a reasonable way to think about the long run response of durables to aggregate shocks.

In the subsequent paper, Caballero (1993) argued that the slow adjustment could reflect the infrequent and lumpy microeconomic purchases of consumer durables due to the adjustment costs involved in purchasing durable goods. A convex adjustment cost specification in which a quadratic cost of adjustment enters the utility function has been frequently adopted with the representative agent facing adjustment costs (Bernanke, 1984). The convex adjustment cost specification, however, implies that the representative consumer optimally adjusts in small amounts upon all innovations. The convex adjustment cost specification thus apparently disregards the typical observation that consumers purchase durables in lump-sums and infrequently. A representative agent model incorporating intermittent adjustment, however, cannot be applied directly to the aggregate time-series data, which is typically continuous. The time aggregation problem arises in this case due to the heterogeneous nature of the intermittent adjustments by different consumers.

Cabellero (1993) dealt with this aggregation problem by shifting the focus away from the rational expectations optimization framework and developing a framework in the context of (S, s) inventory model in which a dynamic analysis of the cross sectional distribution of the durable good stock is made operational. He decomposed the durable good stock into a target component and a departure variable and showed that an individual upgrades the durable stock only when the departure variable reaches the lower trigger point s. The target component is determined by the frictionless model; thus, the actual stock can be considered as the sum of the durable stock predicted by the frictionless model and the noise term.

Caballero (1993) shows that the problem of describing the dynamic behavior of aggregate durable purchases can be reduced to that of describing the dynamic behavior of the first moment of the cross sectional density of the departure variable. He also shows that the changes in the mean of the cross sectional density of the departure variable can be expressed in terms of the flow of consumers upgrading (and downgrading) their stock. The path of the mean of the departure variable, which summarizes the difference between the frictionless and actual aggregate paths of the durable stock, depends on the size of the increase in the durable stock of those who decide to upgrade times the fraction of units that upgrade their stock and also by the aggregate uncertainty faced by each unit among other factors. The expression for the path of the aggregate durable stock can be obtained by adding the path of the frictionless stock of durables to that of the first moment of the cross sectional density. The time series implication of the dynamic framework is that the durable expenditure is a sum of a value predicted by the frictionless RE-PIH model and a noise factor associated with a nonconvex adjustment. Thus the framework allows one to explain the time-series departure between the actual durable expenditures and the durable expenditures predicted by frictionless RE-PIH. The expenditures on furnitures are shown to display larger departure from the frictionless RE-PIH model than the expenditures on cars. Within the automobile expenditures, the 1970's are shown to display larger departure from the frictionless case than other periods.

2.2. Issues

The standard RE-PIH model of durable goods implicitly assumes that consumers incur durable expenditures every period. The standard stock-flow identity implies that in every period consumers replace the depreciated portion of the stock last period with new durables purchases this period. However, as pointed out by Caballero, in the real world consumers tend to purchase durables in lump-sums and infrequently. Thus, whether the standard stock-flow identity is a reasonable description of a representative consumer's durables purchase behavioral pattern is questionable. If the durable expenditure interval is longer than one period, then simply using the standard stock-flow identity to derive the durables expenditures model may result in a spurious model that does not correctly reflect the representative consumer's behavior. Bar-Ilan and Blinder (1992) argue against the typical model assumption that consumers optimize upon an innovation every period. They claim that inertial behavior is a pervasive fact of economic life and is not inconsistent with rational behavior. Grossman and Laroque (1990) also showed that optimal consumption is not a smooth function of wealth and it is optimal for consumers to wait until a large change in wealth occurs before adjusting their consumption.

In this paper, the standard stock flow identity will be modified to reflect the infrequent discontinous purchases of durables. The discontinuous micro-level representative agent model, however, cannot be used directly to test the aggregate time-series data of durables expenditures due to the heterogeneous nature of the intermittent adjustments by different consumers. The issue is how to reconcile the discrepancy between the continuous aggregate time-series data and the typical microeconomic observation of infrequent discontinuous purchases of durables.

Caballero (1993) has dealt with the issue above by abandoning the rational expectations optimization framework and developing a threshold adjustment rule framework in which a dynamic aggregation of stochastically heterogeneous units is made operational. This paper takes a different approach: the connection between the discontinuous micro-level model and continuous aggregate data is made by explicitly aggregating the microeconomic optimization outcome across the data sampling interval. The framework proposed in this paper assumes that a representative consumer receives income news every month with a durables purchase interval that is longer than one month to reflect the fact that durable expenditures are made infrequently. The connection between discontinuous microeconomic actions and continuous aggregate expenditures is made by explicitly aggregating the micro-level optimization outcome with the additional assumptions that (i) the consumers' durable goods purchase intervals are identical and that (ii) the consumer population is evenly spread out among subgroups of consumers making expenditures at different months. This approach not only allows one to circumvent the complications arising from dealing with the cross-sectional aggregation problem, but also provides a tractable way to address the time aggregation issue within the representative agent framework by assuming homogeneous consumers.

The standard RE-PIH models also impose a strong intertemporal separability assumption. In these models, preferences are assumed to be time separable. Mankiw (1982) questioned this restriction placed on the utility function as one possible source of the model's failure. Heaton (1993) argued that temporal aggregation and time-nonseparable preferences can interact in an important way. He classified time non-separable preferences into local substitution and habit persistence effects. Local substitution arises due to the durability of consumption. If one distinguishes between time of buying and time of using, then current consumption is not only a function of current expenditure but also of past expenditures. Local substitution implies that the coefficient on lagged consumption is positive. Heaton goes on to argue that consumption is locally substitutable in the short run but that in the long run, habits will form slowly. So with low frequency data such as quarterly or annual data, a habit effect could dominate the durability effect of local substitution. Ferson and Constantinides (1991) also found that the habit persistence effect dominates local substitution at quarterly and annual frequencies.

Heaton (1993) also pointed out that there is strong evidence for habit persistence that forms over the flow of services from durables. It is reasonable to think that the habit effect is more pronounced with durables than it is with nondurables. Habits can persist such that an individual who consumes a lot in period t-1 will get used to that high level of consumption and will want to consume more in period t. Consumers will want to replace old durables with new durables that give better performance or utility. This paper presents a RE-PIH model incorporating time non-separable preferences in the form of habit persistence. The time-series implication of relaxing the strong intertemporal separability assumption is studied in the following chapter.

Empirical analysis of the RE-PIH on durable goods has been conducted with seasonally adjusted data without questioning its validity. Many researchers, however, have questioned the use of seasonally adjusted data. Newbold and Bos (1990), for example, claimed that the seasonal adjustment procedures currently widely in use are essentially ad hoc because they are developed on the basis of intuitive plausibility and experience. Bell and Hilmer (1984) suggested that researchers should be concerned that the benefits from using simplified seasonally adjusted data not be outweighed by the cost of distortion induced.

The possible bias that could be induced by using seasonally adjusted data made some researchers favor modeling seasonality directly using the seasonally unadjusted data rather than using seasonally adjusted data. Plosser (1978), for example, contends that incorporating seasonality directly in the model not only provides a researcher with a better understanding of the source and type of seasonal variation, but also eliminates concerns of any distortions the seasonal adjustment procedure may have in analyzing and interpreting the model. Bell and

Hilmer (1984) also show that often the seasonal and ARMA coefficients are best identified and estimated jointly. These researchers contend that the seasonal adjustment process can lead to loss of valuable information resulting in a rejection of a true data generating model. Miron (1986), for example, argued that seasonal fluctuation is likely to be well described by a rational expectation model because agents will anticipate the fluctuation and will adjust their behavior accordingly. Hence, using seasonally adjusted data to model behavior of such agents can reach a biased conclusion about the business cycle fluctuation in consumption. Sims (1993) also states that in planning their consumption behavior, rational agents will take account of seasonal fluctuation and modeling such agents using seasonally adjusted data could severely bias the outcome by throwing away valuable information. In this paper, the base model is extended to incorporate stochastic seasonality. The time series representation of the model that incorporates the seasonal variation is presented. The seasonally unadjusted data is applied to the stochastic seasonal model.

THEORETICAL FRAMEWORK

3.1. Base Model

Model assumptions:

1. Consumers are assumed to receive income news on a monthly basis. A monthly frequency is also consistent with the frequency at which workers typically receive wage income and adjust their permanent incomes.

2. The consumer durable goods purchase interval, however, is assumed to be longer than one month. This is to reflect the fact that consumer durables expenditures behavior involves infrequent lump-sum purchases. Specifically, consumers are assumed to make lump-sum purchases of durable goods every i months.

3. No adjustment costs.

4. Time-separable preferences.

5. Quadratic utility function.

Let the durable purchase interval be i periods (months). The accelerated depreciation method of declining balance (or geometric) depreciation is applied each period such that the depreciation rate is applied to the undepreciated value of the old durables from the previous period. Thus, the remaining value of the old durables after i periods is $(1-\delta)^i K_{t-i}$, where K_{t-i} is the durable stock at period t-i and δ is the depreciation rate. Hulten and Wykoff (1981) showed that many studies obtained the result that the depreciation is accelerated and well approximated by a geometric depreciation schedule.

The stock-flow identity can then be modified as

$$K_t = (1-\delta)^i K_{t-i} + C_t \tag{3.1}$$

Consider the standard model of durable consumption under uncertainty with the modified stock-flow identity to reflect the multi-period durable goods purchase interval. Since the representative agent is assumed to make expenditures on durable goods every i months, the optimization problem can be written as follows.

The representative agent maximizes with respect to K_{t+j}, j = 0,i,2i,...,T:

$$E_t \sum_{s=0}^{T-t} (1+\theta)^{-s} U(K_{t+s})$$

subject to

$$\{K_t - (1-\delta)^i K_{t-i} - \sum_{h=0}^{i-1}(1+r)^h Y_{t-h}\} + (1+r)^{-i}\{K_{t+i} - (1-\delta)^i K_t - \sum_{h=0}^{i-1}(1+r)^h Y_{t+i-h}\}$$
$$+ \ldots + (1+r)^{-(T-t)}\{K_T - (1-\delta)^i K_{T-i} - \sum_{h=0}^{i-1}(1+r)^h Y_{T-h}\} = W_t$$

where,

E_t = the mathematical expectation conditional on all information available in time t

θ = rate of subjective time preference

r = real rate of interest, assumed constant over time

U( ) = strictly concave one-period utility function

K_t = stock of durable goods

Y_t = earnings, the only source of uncertainty

W_t = asset apart from human capital

δ = depreciation rate of the consumer's durable stock.

Solving the optimization problem gives the Euler equation (see Appendix A);

$$E_t U'(K_{t+i}) = [(1+\theta)/(1+r)]^{i} \, U'(K_t) \qquad (3.2)$$

If the utility function is quadratic, then K_t follows an AR (|| i ||) process (see Appendix A);

$$K_t = a_0 + a_1 K_{t-i} + \mu_t \qquad (3.3)$$

where, t = t, t + i, t + 2i, ..., T

μ_t = income news (innovation) consumer receives since the last purchase period.

According to Hall (1978), the disturbance term μ_t summarizes the impact of all new information regarding consumer's lifetime well-being that becomes available in period t.

Proposition 1: If utility is quadratic and the modified stock-flow identity is applied, the lag change in durable expenditures C_t will follow an MA (|| i ||) process rather than the MA (1) process suggested by Mankiw. That is,

$$(1- L^i)C_t = a + \mu_t - (1-\delta)^i \mu_{t-i} \qquad (3.4)$$

where L = lag operator.

Proof:

Substitute equation (3.3) into equation (3.1) and rearrange to get

$C_t = a + a_1 C_{t-i} + \mu_t - (1-\delta)^i \mu_{t-i}$

where $a = a_0(1- (1-\delta)^i)$ and $a_1 = [(1+ \theta)/(1+ r)]^i$

Assume $\theta = r$. Then, (3.4) follows.

According to the representative agent model, if the durable purchase interval is i periods, the lag change in durables expenditures at time t is $(1-L^i)C_t =$ constant $+ \mu_t + \beta\, \mu_{t-i}$, where $\beta = - (1-\delta)^i$. The discontinuous representative agent model, however, cannot be used directly to estimate the aggregate time-series data of durable expenditures. The time aggregation problem has to be explicitly addressed in this case because different consumers will adjust their durable stocks in different periods. This paper addresses the time aggregation problem by explicitly aggregating the microeconomic optimization outcome across the quarterly aggregated data sampling interval. Additional assumptions required for the explicit aggregation are listed below.

Additional assumptions:

6. Let ε_t = income news consumers receive at month t assumed to be invariant among consumers and serially uncorrelated with zero mean and constant variance.

7. Monthly interest on income innovation is insignificant, assumed to be zero.

8. Length of the durable expenditures interval is identical for all consumers.

9. Consumer population is evenly spread out among subgroups of consumers purchasing durables at different months.

Then the change in durables expenditures for the group adjusting durables in month t is

$$(1-L^i)C_t = \mu_t + \beta\, \mu_{t-i} \qquad (3.5)$$

where, $\mu_t = (1+L+L^2+ \ldots + L^{i-1})\varepsilon_t$,

that is, the sum of the income news since the last purchase period.

$\therefore (1-L^i)C_t = \mu_t + \beta\mu_{t-i} = (1+\beta L^i)\mu_t = (1+\beta L^i)(1+L+L^2+\ldots+L^{i-1})\varepsilon_t$

take first difference on both sides;

$(1-L)(1-L^i)C_t = (1+\beta L^i)(1-L)(1+L+L^2+\ldots+L^{i-1})\varepsilon_t$

$\therefore (1-L^i)(1-L)\,C_t = (1-L^i)(1+\beta L^i)\,\varepsilon_t$

$(1-L^i)$ could be cancelled out from both sides of the equation to yield following equation;

$$(1-L)C_t = (1+\beta L^i)\varepsilon_t \tag{3.6}$$

According to equation (3.6), change in durable consumption expenditures is a function of the expenditure interval i. Since the model in equation (3.6) is in monthly frequency, explicit aggregation is performed across a quarter to examine the quarterly aggregated time-series properties. The monthly and quarterly aggregated changes in durable goods expenditures are shown in Table 3.1. For example, if i = 3 (one quarter), then

$\Delta C_1^q = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \beta(\varepsilon_{-2} + \varepsilon_{-1} + \varepsilon_0)$

$\Delta C_2^q = \varepsilon_4 + \varepsilon_5 + \varepsilon_6 + \beta(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$

$\Delta C_3^q = \varepsilon_7 + \varepsilon_8 + \varepsilon_9 + \beta(\varepsilon_4 + \varepsilon_5 + \varepsilon_6)$

$\text{Var}(\Delta C_t^q) = 3(1+\beta^2)\sigma_\varepsilon^2$

$\text{Cov}(\Delta C_1^q, \Delta C_2^q) = 3\beta\sigma_\varepsilon^2$

$\text{Cov}(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 3, 4, 5, \ldots$

$\text{Corr}(\Delta C_1^q, \Delta C_2^q) = \beta/(1+\beta^2)$

$\text{Corr}(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 3, 4, 5, \ldots$

Table 3.1. Quarterly aggregate changes with variable purchase intervals

$(1\text{-}L)C_1^m = \varepsilon_1 + \beta\varepsilon_{1-i}$
$(1\text{-}L)C_2^m = \varepsilon_2 + \beta\varepsilon_{2-i}$
$(1\text{-}L)C_3^m = \varepsilon_3 + \beta\varepsilon_{3-i}$
.
.
$\Delta C_1^q = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \beta(\varepsilon_{1-i} + \varepsilon_{2-i} + \varepsilon_{3-i})$
$\Delta C_2^q = \varepsilon_4 + \varepsilon_5 + \varepsilon_6 + \beta(\varepsilon_{4-i} + \varepsilon_{5-i} + \varepsilon_{6-i})$
$\Delta C_3^q = \varepsilon_7 + \varepsilon_8 + \varepsilon_9 + \beta(\varepsilon_{7-i} + \varepsilon_{8-i} + \varepsilon_{9-i})$
.
.

This is the case of the standard RE-PIH model proposed by Mankiw (1982) where the durable goods purchase interval is equal to the quarterly data sampling interval. Thus, the standard model can be considered as a special case of the variable purchase interval model. Examination of the quarterly series gives the following results (Appendix B).

If $i = 1$; then $\mathrm{Corr}(\Delta C_1^q, \Delta C_2^q) = \beta / \{3(1+\beta^2) + 4\beta\}$ ... MA (1)

If $i = 2$; then $\mathrm{Corr}(\Delta C_1^q, \Delta C_2^q) = 2\beta / \{3(1+\beta^2) + 2\beta\}$ ... MA (1)

where $\beta = -(1-\delta)^i$

If $i = 3j$; $j=1,2,\ldots$, then $\mathrm{Corr}(\Delta C_1^q, \Delta C_{1+j}^q) = \beta / (1+\beta^2)$ ... MA ($\|j\|$)

where $\beta = -(1-\delta)^{3j}$

If $i = 3j + k$; ($j=1,2,\ldots$; $k=1,2,\ldots$) then

$$Corr\ (\Delta C_1^q, \Delta C_{1+j}^q) = (3-k)\beta / 3(1+\beta^2)$$

$$Corr\ (\Delta C_1^q, \Delta C_{2+j}^q) = k\beta / 3(1+\beta^2) \qquad \ldots MA\ (\|j, j+1\|)$$

where $\beta = -(1-\delta)^{3j+k}$

Table 3.2 presents autocorrelation coefficients and MA coefficients for different purchase intervals assuming monthly depreciation rate of 1.7 % equivalent to the quarterly depreciation of 5 %. For example, when the purchase interval is 12 months, the change in consumption expenditures should approximate an MA (|| 4 ||) process with the MA coefficient of -0.8. Mankiw's (1982) original model is the case when i = 3, where the MA coefficient equals -0.95 and the related first order autocorrelation is –0.5. The time series implication of the RE-PIH model on durable consumption is thus dependent on the purchase interval of durable goods.

Table 3.2. Moving average model with variable purchase intervals

interval: i months	model	autocorrelation	MA coefficient
1	MA (1)	-0.498	-
2	MA (1)	-0.498	-
3	MA (1)	-0.498	-0.95
4	MA (2)	-0.33, -0.17	-
5	MA (2)	-0.17, -0.33	-
6	MA (|| 2 ||)	-0.497	-0.90
12	MA (|| 4 ||)	-0.489	-0.81
24	MA (|| 8 ||)	-0.460	-0.66
36	MA (||12||)	-0.418	-0.54
48	MA (||16||)	-0.368	-0.44
60	MA (||20||)	-0.357	-0.32
72	MA (||24||)	-0.291	-0.27

With aggregate data, the change in aggregate durable expenditures is a function of average purchase interval of durable goods. Thus the base model allows one to identify the average purchase interval of durable goods.

The base model result turns out to be insensitive to the inclusion of the monthly interest on income innovations. The aggregate quarterly changes (Table 3.1) when monthly interest on income innovation is incorporated into the base model are shown below;

$$\Delta C_1^q = \{(1+r)^2\varepsilon_1 + (1+r)\varepsilon_2 + \varepsilon_3\} + \beta\{(1+r)^2\varepsilon_{1-i} + (1+r)\varepsilon_{2-i} + \varepsilon_{3-i}\}$$
$$\Delta C_2^q = \{(1+r)^2\varepsilon_4 + (1+r)\varepsilon_5 + \varepsilon_6\} + \beta\{(1+r)^2\varepsilon_{4-i} + (1+r)\varepsilon_{5-i} + \varepsilon_{6-i}\}$$
$$\Delta C_3^q = \{(1+r)^2\varepsilon_7 + (1+r)\varepsilon_8 + \varepsilon_9\} + \beta\{(1+r)^2\varepsilon_{7-i} + (1+r)\varepsilon_{8-i} + \varepsilon_{9-i}\}$$

For example, if the purchase interval i is 3 months, then

$$\Delta C_1^q = \{(1+r)^2\varepsilon_1 + (1+r)\varepsilon_2 + \varepsilon_3\} + \beta\{(1+r)^2\varepsilon_{-2} + (1+r)\varepsilon_{-1} + \varepsilon_0\}$$
$$\Delta C_2^q = \{(1+r)^2\varepsilon_4 + (1+r)\varepsilon_5 + \varepsilon_6\} + \beta\{(1+r)^2\varepsilon_1 + (1+r)\varepsilon_2 + \varepsilon_3\}$$
$$\Delta C_3^q = \{(1+r)^2\varepsilon_7 + (1+r)\varepsilon_8 + \varepsilon_9\} + \beta\{(1+r)^2\varepsilon_4 + (1+r)\varepsilon_5 + \varepsilon_6\}$$

Examination of the series with monthly interest on the income innovations gives the following variance/covariance/correlation results.

$$\mathrm{Var}\,(\Delta C_t^q) = (1+\beta^2)\{1+(1+r)^2+(1+r)^4\}\sigma_\varepsilon^2$$

$$\mathrm{Cov}\,(\Delta C_1^q, \Delta C_2^q) = \beta\{1+(1+r)^2+(1+r)^4\}\sigma_\varepsilon^2$$

$$\mathrm{Cov}\,(\Delta C_1^q, \Delta C_k^q) = 0, \text{ if } k = 3,4,5,\ldots$$

$$\mathrm{Corr}\,(\Delta C_1^q, \Delta C_2^q) = \beta/(1+\beta^2)$$

$$\mathrm{Corr}\,(\Delta C_1^q, \Delta C_k^q) = 0 \text{ if } k = 3, 4, 5, \ldots$$

The correlation result is exactly the same as the base model case when the purchase interval is 3 months. The result holds for different purchase intervals. For example, if the purchase interval is assumed to be 4 months, then

$\Delta C_1^q = \{(1+r)^2\varepsilon_1 + (1+r)\varepsilon_2 + \varepsilon_3\} + \beta\{(1+r)^2\varepsilon_{-3} + (1+r)\varepsilon_{-2} + \varepsilon_{-1}\}$

$\Delta C_2^q = \{(1+r)^2\varepsilon_4 + (1+r)\varepsilon_5 + \varepsilon_6\} + \beta\{(1+r)^2\varepsilon_0 + (1+r)\varepsilon_1 + \varepsilon_2\}$

$\Delta C_3^q = \{(1+r)^2\varepsilon_7 + (1+r)\varepsilon_8 + \varepsilon_9\} + \beta\{(1+r)^2\varepsilon_3 + (1+r)\varepsilon_4 + \varepsilon_5\}$

the following variance/covariance/correlation results:

$\text{Var}(\Delta C_t^q) = (1+\beta^2)\{1+(1+r)^2+(1+r)^4\}\sigma_\varepsilon^2$

$\text{Cov}(\Delta C_1^q, \Delta C_2^q) = \beta\{(1+r)^3+(1+r)\}\sigma_\varepsilon^2$

$\text{Cov}(\Delta C_1^q, \Delta C_3^q) = \beta(1+r)^2\sigma_\varepsilon^2$

$\text{Cov}(\Delta C_1^q, \Delta C_k^q) = 0$, if $k = 4, 5, 6\ldots$

$\text{Corr}(\Delta C_1^q, \Delta C_2^q) = \{\beta/(1+\beta^2)\}[\{(1+r)^3+(1+r)\}/\{1+(1+r)^2+(1+r)^4\}]$

$\text{Corr}(\Delta C_1^q, \Delta C_3^q) = \{\beta/(1+\beta^2)\}[(1+r)^2/\{1+(1+r)^2+(1+r)^4\}]$

$\text{Corr}(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 4, 5, 6 \ldots$

If we choose a monthly interest rate r = 0.0025 (0.25%), equivalent to an annual real interest rate of 3 % then,

$\text{Corr}(\Delta C_1^q, \Delta C_2^q) = 2\beta/3(1+\beta^2)$

$\text{Corr}(\Delta C_1^q, \Delta C_3^q) = \beta/3(1+\beta^2)$

The correlation result is exactly the same as the base model case when the purchase interval is 4 months. The correlation results are insensitive to the choice of different monthly interest rate. Calculation using monthly interest rate range of 0.083% ~ 1%, equivalent to annual rate of 1% ~ 12%, gives the same correlation results. The analysis shows that the base model implications are insensitive to the choice of different monthly interest rate on the income innovations. In the following sections, I will examine the time-series implication when some of the restrictions of the base model are relaxed.

3.2. Habit Persistence Model

The base model assumption that preferences are time-separable is relaxed to allow for time non-separable preference structure in the form of habit persistence. Consider the standard representative consumer optimization problem except that the utility function is specified as a habit persistence preference. A simple way to model this habit persistence is to include the lagged consumption variable in the current utility function. The coefficient on the lagged consumption should be negative so that the current period marginal utility of lagged consumption is negative. With durable goods, a stock variable K replaces the flow C in the utility function. Thus, in the context of the multi-period durable goods purchase interval model, the lagged durable stock K_{t-i} enters the utility function with a negative sign. Intuitively, an individual who consumes a lot in period t-i will get used to that high level of consumption and will want to consume more in period t.

Consider the durable consumption model under uncertainty with the habit persistence preference structure.

The representative agent maximizes with respect to K_{t+j} , $j = 0, i, 2i, \ldots, T$:

$$E_t \sum_{s=0}^{T-t} (1+\theta)^{-s} U(K_{t+s} - \phi K_{t+s-i})$$

subject to

$$\{K_t - (1-\delta)^i K_{t-i} - \sum_{h=0}^{t-1} (1+r)^h Y_{t-h}\} + (1+r)^{-i} \{K_{t+i} - (1-\delta)^i K_t - \sum_{h=0}^{t-1} (1+r)^h Y_{t+i-h}\}$$
$$+ \ldots + (1+r)^{-(T-t)} \{K_T - (1-\delta)^i K_{T-i} - \sum_{h=0}^{t-1} (1+r)^h Y_{T-h}\} = W_t$$

where

E_t = the mathematical expectation conditional on all information available in time t

θ = rate of subjective time preference

r = real rate of interest, assumed constant over time

U( ) = strictly concave one-period utility function

K_t = stock of durable goods

Y_t = earnings, the only source of uncertainty

W_t = asset apart from human capital

δ = depreciation rate of the consumer's durable stock.

ϕ = subjective habit persistence parameter

Solving the optimization problem gives the following Euler equation (see Appendix C):

$$E_t U'(K_{t+i} - \phi K_t) = [(1+\beta)^i / \{(1+r)^i + \phi\}]\ U'(K_t - \phi K_{t-i}) \qquad (3.7)$$

The Euler equation indicates that the rational consumer attempts to keep the discounted marginal utility of consumption equalized across time.

Proposition 2: If the utility function is quadratic, then with the habit persistence specification, the lag change in durable expenditures should follow an ARMA (1, || i ||) process;

$$(1-L^i)C_t = \phi(1-L^i)C_{t-1} + \mu_t - (1-\delta)^i\mu_{t-i} \quad (3.8)$$

Proof:

From equation (3.7), assume $(1+\beta)^i = (1+r)^i + \phi$

and quadratic utility; $U(\,) = -0.5(K - (K_t - \phi K_{t-i}))^2$.

Substitute into the Euler equation (3.7) to get

$E_t(K - (K_{t+i} - \phi K_t)) = K - (K_t - \phi K_{t-i})$

Rearranging the equation gives

$K_t = (1+\phi)K_{t-i} - \phi K_{t-2i} + \mu_t$.

Substitute the equation into equation (3.1) to get

$(1-L^i)C_t = \phi(1-L^i)C_{t-1} + \mu_t - (1-\delta)^i\mu_{t-i}$

As in the base model case, the time-aggregation problem is addressed by explicitly aggregating the microeconomic optimization outcome across the quarterly data sampling interval. The same additional assumptions will apply in this case as in the base model case. Then the change in durables expenditures for the group adjusting durables in month t is

$$(1-L^i)C_t = \phi(1-L^i)C_{t-1} + (1+\beta L^i)\mu_t \tag{3.9}$$

where, $\mu_t = (1+L+L^2+ \dots + L^{i-1})\varepsilon_t$, that is, the sum of the income news since the last purchase period.

$\therefore (1-L^i)C_t = \phi(1-L^i)C_{t-1} + (1+\beta L^i)(1+L+L^2+ \dots + L^{i-1})\varepsilon_t$

take first difference on both sides;

$\therefore (1-L)(1-L^i)C_t = \phi(1-L)(1-L^i)C_{t-1} + (1+\beta L^i)(1-L)(1+L+L^2+ \dots + L^{i-1})\varepsilon_t$

$\therefore (1-L)(1-L^i)C_t = \phi(1-L)(1-L^i)C_{t-1} + (1+\beta L^i)(1-L^i)\varepsilon_t$

$(1-L^i)$ could be cancelled out from both sides of the equation resulting in the following model:

$$(1-L)C_t = \phi(1-L)C_{t-1} + (1+\beta L^i)\varepsilon_t \tag{3.10}$$

To examine the quarterly aggregated time-series property of the ARMA (1, || i ||) model, explicit aggregation is performed across the quarters (Table 3.3).

Table 3.3. Quarterly aggregate changes of habit persistence model

$(1-L)C_1^m = \phi(1-L)C_0^m + \varepsilon_1 + \beta\varepsilon_{1-i}$
$(1-L)C_2^m = \phi(1-L)C_1^m + \varepsilon_2 + \beta\varepsilon_{2-i}$
$(1-L)C_3^m = \phi(1-L)C_2^m + \varepsilon_3 + \beta\varepsilon_{3-i}$

.

.

$\Delta C_1^q = \phi\Delta C_0^q + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) + \beta(\varepsilon_{1-i} + \varepsilon_{2-i} + \varepsilon_{3-i})$
$\Delta C_2^q = \phi\Delta C_1^q + (\varepsilon_4 + \varepsilon_5 + \varepsilon_6) + \beta(\varepsilon_{4-i} + \varepsilon_{5-i} + \varepsilon_{6-i})$
$\Delta C_3^q = \phi\Delta C_2^q + (\varepsilon_7 + \varepsilon_8 + \varepsilon_9) + \beta(\varepsilon_{7-i} + \varepsilon_{8-i} + \varepsilon_{9-i})$

.

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Because of the additional AR (1) terms involved in the equations, the identification of the model using the autocorrelation functions becomes more difficult. However, if the expenditures interval i is equal to multiples of 3 months then the model is readily identifiable. For example if i = 3, then;

$$\Delta C_1^q = \phi \Delta C_0^q + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) + \beta(\varepsilon_{-2} + \varepsilon_{-1} + \varepsilon_0) = \phi \Delta C_0^q + \mu_1 + \beta\mu_0$$
$$\Delta C_2^q = \phi \Delta C_1^q + (\varepsilon_4 + \varepsilon_5 + \varepsilon_6) + \beta(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) = \phi \Delta C_1^q + \mu_2 + \beta\mu_1$$
$$\Delta C_3^q = \phi \Delta C_2^q + (\varepsilon_7 + \varepsilon_8 + \varepsilon_9) + \beta(\varepsilon_4 + \varepsilon_5 + \varepsilon_6) = \phi \Delta C_2^q + \mu_3 + \beta\mu_2$$

where μ_j is sum of the monthly income innovation during the quarter j; $\mu_j = \varepsilon_1^j + \varepsilon_2^j + \varepsilon_3^j$.
If i = 6, then;

$$\Delta C_1^q = \phi \Delta C_0^q + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) + \beta(\varepsilon_{-5} + \varepsilon_{-4} + \varepsilon_{-3}) = \phi \Delta C_0^q + \mu_1 + \beta\mu_{-1}$$
$$\Delta C_2^q = \phi \Delta C_1^q + (\varepsilon_4 + \varepsilon_5 + \varepsilon_6) + \beta(\varepsilon_{-2} + \varepsilon_{-1} + \varepsilon_0) = \phi \Delta C_1^q + \mu_2 + \beta\mu_0$$
$$\Delta C_3^q = \phi \Delta C_2^q + (\varepsilon_7 + \varepsilon_8 + \varepsilon_9) + \beta(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) = \phi \Delta C_2^q + \mu_3 + \beta\mu_1$$

If i = 12, then;

$$\Delta C_1^q = \phi \Delta C_0^q + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) + \beta(\varepsilon_{-11} + \varepsilon_{-10} + \varepsilon_{-9}) = \phi \Delta C_0^q + \mu_1 + \beta\mu_{-3}$$
$$\Delta C_2^q = \phi \Delta C_1^q + (\varepsilon_4 + \varepsilon_5 + \varepsilon_6) + \beta(\varepsilon_{-8} + \varepsilon_{-7} + \varepsilon_{-6}) = \phi \Delta C_1^q + \mu_2 + \beta\mu_{-2}$$
$$\Delta C_3^q = \phi \Delta C_2^q + (\varepsilon_7 + \varepsilon_8 + \varepsilon_9) + \beta(\varepsilon_{-5} + \varepsilon_{-4} + \varepsilon_{-3}) = \phi \Delta C_2^q + \mu_3 + \beta\mu_{-1}$$
$$\Delta C_4^q = \phi \Delta C_3^q + (\varepsilon_4 + \varepsilon_5 + \varepsilon_6) + \beta(\varepsilon_{-2} + \varepsilon_{-1} + \varepsilon_0) = \phi \Delta C_3^q + \mu_4 + \beta\mu_0$$
$$\Delta C_5^q = \phi \Delta C_4^q + (\varepsilon_7 + \varepsilon_8 + \varepsilon_9) + \beta(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) = \phi \Delta C_4^q + \mu_5 + \beta\mu_1$$

The analysis shows that with a simple habit persistence preference structure, the change in quarterly aggregate durable consumption expenditures should exhibit an ARMA (1, || j ||) process when the expenditures interval i = 3j, where j is the expenditures interval in number of quarters. The autoregressive (AR) coefficient is equal to the habit persistence parameter ϕ and the moving average (MA) coefficient is equal to the depreciation term β.

3.3. Stochastic seasonal model

Seasonality is a cyclical behavior of known periodicity. If C_t has a fixed seasonal pattern of period d, then $C_{t-d} = C_t = C_{t+d} = \ldots$ and thus $(1- L^d)C_t = C_t - C_{t-d} = 0$, where L is the lag operator. If we want to model stochastic seasonality, the simplest form would be to allow seasonal pattern to change at random, then $(1- L^d)C_t = C_t - C_{t-d} = \varepsilon_t$, where ε_t is a white noise process. The argument could be extended to allow for seasonal pattern to change with some known pattern such as a moving average of known order q. In this paper, one additional step is taken where the seasonal change in durable goods expenditures is related to seasonal change in income savings μ_t in such a way to yield the following relationship;

$$(1- L^d)C_t = \mu_t - (1- \delta)^d \mu_{t-d} = (1 - (1- \delta)^d L^d)\mu_t . \qquad (3.11)$$

This model is equivalent to the base model (equation 3.5.) with periodicity d replacing the expenditure interval i. Thus, one can derive equation (3.11) from a representative agent optimization framework of RE-PIH. The relationship displays a seasonal difference in consumption for the same period in two consecutive years. It would be reasonable to assume that this relationship will hold for each period, that is,

$$(1- L^d)C_{t-1} = (1 - (1- \delta)^d L^d)\mu_{t-1} .$$

It is also quite plausible that the error component μ_t will be related to μ_{t-1} and so on. To model such period-to-period relationship, the following second model is introduced;

$$(1\text{-}L)\mu_t = (1\text{-}L)\{\varepsilon_t + \varepsilon_{t-1} + \ldots + \varepsilon_{t-(d-1)}\} \tag{3.12}$$

where ε_t is the income innovation at period t assumed to be a white noise process. Substituting equation (3.12) into (3.11) yield the following stochastic seasonal model;

$$(1\text{-}L^d)(1\text{-}L)C_t = (1-(1\text{-}\delta)^d L^d)(1\text{-}L)(1+L+\ldots+L^{d-1})\varepsilon_t = (1-(1\text{-}\delta)^d L^d)(1\text{-}L^d)\varepsilon_t$$

$$\therefore (1\text{-}L^d)(1\text{-}L)C_t = (1-(1\text{-}\delta)^d L^d)(1\text{-}L^d)\varepsilon_t \tag{3.13}$$

The stochastic seasonal model (3.13) can be interpreted as a seasonal difference in consumption change for the same period in two consecutive years. The model is essentially the multiplicative seasonal model or the Seasonal ARIMA model that is widely used in estimating seasonal variations in time-series analysis. Thus the analysis shows how a model incorporating seasonal variation could be derived from a rational expectations optimization framework. The stochastic seasonal model assumes that a series contains seasonal unit roots. The seasonal difference filter $(1\text{-}L^d)$ can be cancelled out from both sides of the equation to yield the following residual model.

$$(1\text{-}L)C_t = (1-(1\text{-}\delta)^d L^d)\varepsilon_t$$

If a quarterly series is to be estimated, the period d equals 4, and the residual model predicts a MA (|| 4 ||) process. The MA (|| 4 ||) coefficient should approximate –0.81 according to the base model analysis when the seasonal period is 12 months or 4 quarters.

4. DATA AND ESTIMATION METHOD

4.1. Data

The durable goods expenditures data from the U.S. National Income and Product Accounts published by the Bureau of Economics Analysis of the Department of Commerce are used for the estimation. The NIPA provides seasonally adjusted and seasonally unadjusted quarterly data for durable goods and two sub-categories. The two subcategories are i) motor vehicles and parts and ii) furniture and household equipment. Personal consumption expenditures on motor vehicles and parts includes purchases of new autos and net purchases of used autos plus expenditures on tires, tubes, accessories and other parts. Personal consumption expenditures on furniture and household equipment includes furniture, kitchen, and other household appliances, video and audio products and computing equipment. For the seasonally adjusted data, the per capita series is constructed by dividing the real expenditures series by the total population in the mid-point of the period. The per capita series is in 1992 dollars. The sample period used is 1959:3 to 1995:4. The NIPA, however, only provides seasonally unadjusted data on nominal consumption expenditures. Thus, the real expenditures series has to be constructed by deflating the nominal series by a price deflator. The Bureau of Labor Statistics publishes seasonally unadjusted consumer price index on the detailed component of the durable goods expenditures. To construct a real expenditures series, the nominal expenditures series are deflated by the seasonally unadjusted component of the CPI published by the BLS. The CPI components used to deflate the three nominal expenditures series are durables, new vehicles, and furnitures. These three CPI components are chosen because they most closely match the nominal durables expenditures series and the two subcategories. The base period used is 1982-1984. The real series is then divided by the total population in the mid-point of the period to obtain per capita base data. The sample period used is 1959:3 to 1990:4 due to data availability.

4.2. Estimation Method

Box-Jenkins (1976) method is used to identify and estimate the time-series models. For the seasonally adjusted series, the change in consumption expenditures on durable goods and its sub-categories are fit by Box-Jenkins methods to an ARMA model. Estimated models are then examined to see whether they are consistent with the time-series implications of the multi-period purchase interval models presented in this paper. For the seasonally unadjusted series, the seasonal differencing filter is used to seasonally difference the unadjusted series. The residual series is then first differenced and estimated by a Box-Jenkins method. The Akaike information criterion and the Schwartz Bayesian criterion, which are goodness of fit measures, are used to select among the different models. The Ljung-Box Q-statistics is used for the diagnostic checking of any evidence of statistically significant residual autocorrelations.

The distribution theory underlying the Box-Jenkins approach necessitates the model to be stationary, thus, it is important to work with a series that meets this underlying assumption. The upward trend in the durables expenditures series suggests that the series are nonstationary processes. The nature of the trend in a time-series is an important issue. Simply detrending with a deterministic time trend a series that is actually difference stationary will not yield a stationary series since the stochastic trend is not eliminated. Likewise, differencing a trend stationary series is not appropriate because it introduces a noninvertible unit root process into the moving average component of the model. Nelson and Plosser (1982) demonstrated that many important macroeconomic variables appear to be difference stationary rather than trend stationary. A difference stationary process is also called a unit root process.

To test whether the durable expenditures series are unit root processes, Augmented Dickey-Fuller (ADF) tests are performed. For the seasonally unadjusted series, however, an alternative procedure developed by Hylleberg et al. (1990) is used to test for a seasonal unit roots. Many time-series analysts have applied the seasonal differencing filter $(1-L^4)$ to model seasonality, which implicitly assumes that seasonal nonstationarity is caused by seasonal unit roots. The seasonal difference filter can be written as $(1-L^4) = (1-L)(1+L)(1+L^2)$, which has four roots with modulus one; nonseasonal unit root, semi-annual unit root, and annual unit roots. The use of the seasonal differencing operator, however, is justifiable only when seasonal unit roots are present at all three frequencies. Beaulieu and Miron (1993) warn against mechanical application of the seasonal differencing filter without testing for the presence of seasonal unit roots. Serious misspecification can result if seasonal unit roots are present but not accounted for or if a seasonal differencing filter is used when unit roots are absent at some or all of the seasonal frequencies.

4.2.1. Augmented Dickey- Fuller Test

The distribution theory underlying the Dickey-Fuller test assumes that the errors are serially uncorrelated. Said and Dickey (1984), however, have shown that the Dickey-Fuller test is also applicable when a moving average term is present. The ARIMA (p, 1, q) process is shown to be well approximated by an autoregressive process of order no more than $T^{1/3}$, where T is the total number of observations. The limiting distribution of the statistics produced by estimating coefficients in the autoregressive approximation were shown to be the same as those tabulated by Dickey and Fuller (1976). Therefore it is possible to use a finite-order autoregressive process to approximate the infinite-order autoregression implied by a moving average process.

The following regression equation is used to test for the presence of a unit root.

$$\Delta C_t = \beta_0 + \beta_1 t + \gamma C_{t-1} + \sum_{i=2}^{p} \beta_i \Delta C_{t-i-1} + \varepsilon_t$$

The null hypothesis of the presence of a unit root ($\gamma = 0$) is tested by estimating the above equation and obtaining the estimated value of γ and its standard error to construct a t-statistic. The t-statistic is compared with the critical value τ_τ calculated and reported by Dickey and Fuller (1979). Dickey and Fuller (1981) provide additional F-statistics (ϕ-statistics) for testing the significance of the deterministic time trend under the null of a unit root. To test for the significance of the deterministic time trend term under the null of a unit root (H_0: $\gamma = \beta_1 = 0$), the ϕ_3-statistic is used. According to Said and Dickey (1984), the lag length p should increase with the sample size T and it should not exceed $T^{1/3}$, where T is the total number of observations. Schwert (1987) suggested setting the number of lags p to equal the integer of $4(T/100)^{0.25}$, while Diebold and Nerlove (1990) showed that the integer of $T^{0.25}$ works well in practice.

4.2.2. Seasonal unit root test

Seasonal dummies are frequently used in applied work to deseasonalize time series, which implicitly assumes that seasonality is deterministic and constant. Many macroeconomic time series, however, exhibit seasonal fluctuations that are not constant and appear to evolve over time. The use of the seasonal differencing operator to model seasonality implicitly assumes that seasonality is stochastic and seasonal unit roots are present. Thus, as in the case with detrending, a spurious regression could occur if seasonal dummies are used with seasonally integrated series. The seasonal differencing operator used for the quarterly data can be written as follows: $(1-L^4) = (1-L)(1+L)(1-iL)(1+iL)$, where i corresponds to the imaginary number $(-1)^{1/2}$. Thus the solutions to $(1-L^4) = 0$ are $\{1,-1, i, -i\}$. The unit root 1 is a nonseasonal unit root, while –1, i, and –i are the seasonal unit roots at different frequencies. Hylleberg et al (1990) developed a method to test for the presence of seasonal unit roots in quarterly time series. To illustrate the procedure, suppose $\{C_t\}$ is generated by: $A(L)C_t = \varepsilon_t$ where $A(L) = (1-a_1L)(1+a_2L)(1-a_3iL)(1+a_4iL)$.

If $a_1 = 1$ then one homogeneous solution to $A(L)C_t = \varepsilon_t$ is $(1-L)C_t = 0$. This is the case of a nonseasonal unit root since $\{C_t\}$ repeats itself every period.

If $a_2 = 1$ then one homogeneous solution to $A(L)C_t = \varepsilon_t$ is $C_t = -C_{t-1}$. The $\{C_t\}$ sequence repeats itself every two quarters, which is the case of a semi-annual unit root. To illustrate the point, let $C_t = 1$ then it follows that $C_{t+1} = -1$, $C_{t+2} = 1$, $C_{t+3} = -1$, $C_{t+4} = 1$ ...

If $a_3 = 1$ then one homogeneous solution to $A(L)C_t = \varepsilon_t$ is is $C_t = i\, C_{t+1}$. The $\{C_t\}$ sequence repeats itself every four quarters, which is the case of an annual unit root. To illustrate the point, let $C_t = 1$ then it follows that $C_{t+1} = i$, $C_{t+2} = i^2 = -1$, $C_{t+3} = -\, i$, $C_{t+4} = -\, i^2 = 1 \ldots$

Likewise if $a_4 = 1$ then one homogeneous solution to $A(L)C_t = \varepsilon_t$ is is $C_t = -\, i\, C_{t+1}$. The $\{C_t\}$ sequence repeats itself every four quarters, which is the case of an annual unit root.

To develop the test for seasonal unit roots in quarterly data, let $A(L)C_t = \varepsilon_t$ be a function of a_1, a_2, a_3, and a_4. Take a Taylor series approximation of the polynomial A(L) around the point $a_1 = a_2 = a_3 = a_4 = 1$ and it can be shown that

$$(1-L^4)C_t = \gamma_1(1+L+L^2+L^3)C_{t-1} - \gamma_2(1-L+L^2-L^3)C_{t-1} + (1-L^2)[(\gamma_3 - \gamma_4)i-(\gamma_3 + \gamma_4)L]C_{t-1} + \varepsilon_t$$

where, $\gamma_i = (a_i-1)$.

Define $\gamma_5 = (\gamma_3 - \gamma_4)i$ and $\gamma_6 = (\gamma_3 + \gamma_4)$ then,

$$(1-L^4)C_t = \gamma_1(1+L+L^2+L^3)C_{t-1} - \gamma_2(1-L+L^2-L^3)C_{t-1} + (1-L^2)(\gamma_5 - \gamma_6 L)C_{t-1} + \varepsilon_t$$

To implement the procedure, the following steps are suggested in Enders (1995).

Step 1. Form the following variables:

$$C_{1t-1} = (1+L+L^2+L^3)C_{t-1} = C_{t-1} + C_{t-2} + C_{t-3} + C_{t-4}$$

$$C_{2t-1} = (1-L+L^2-L^3)\, C_{t-1} = C_{t-1} - C_{t-2} + C_{t-3} - C_{t-4}$$

$$C_{3t-1} = (1-L^2)C_{t-1} = C_{t-1} - C_{t-3}$$

Step 2. Estimate the regression:

$$(1-L^4)C_t = \text{constant} + \gamma_1 C_{1t-1} - \gamma_2 C_{2t-1} + \gamma_5\, C_{3t-1} - \gamma_6 C_{3t-2} + \varepsilon_t$$

Step 3. Form the t – statistic for the null hypothesis $\gamma_1 = 0$.

If one does not reject the null $\gamma_1 = 0$, conclude $a_1 = 1$; there is a nonseasonal unit root.

Form the t – statistic for the null hypothesis $\gamma_2 = 0$.

If one does not reject the null $\gamma_2 = 0$, conclude $a_2 = 1$; there is a semi-annual unit root.

Form the F – statistic for the null hypothesis $\gamma_5 = \gamma_6 = 0$.

If one does not reject the null $\gamma_5 = \gamma_6 = 0$; there is an annual unit root.

The critical values are reported in Hylleberg et al (1990).

The above procedure could also be applied when there is a potential moving average (MA) component. Beaulier and Miron (1993) suggest estimating the regression with high-order autoregressive (AR) model to approximate the infinite order AR implied by an MA component, which is an argument based on Said and Dickey (1984).

5. ESTIMATION RESULT

5.1. Estimation with seasonally adjusted data

Figure 5.1 shows the seasonally adjusted series of the consumer durable expenditures, and two subcategories. The consumer durables series reveals an upward trend over the sample period. The furnitures and household appliances series displays a relatively smooth upward trending curve, while the motor vehicle and parts series is much more volatile. The first-differenced series for the three series are presented in Figure 5.2. The first-differenced series of the consumer durable series and the motor vehicle series display constant mean. The trend of the furnitures/appliances series, on the other hand, does not appear to have been removed completely by first differencing, which suggests that the series is either second order integrated process or trend stationary process. To formally test that the consumer durable expenditures series and its subcategories have unit roots, an Augmented Dickey-Fuller (ADF) test is performed. The three series also appear to exhibit relatively larger volatility in the 1980s. One possible way to address the problem would be to take logarithmic transformation of the series to smooth the sequence. But since the models presented in this paper give predictions in level changes, we did not perform any data transformation.

Table 5.1 presents the ADF tests results for durable goods series and two subcategories for the seasonally adjusted data. The regression equation including the intercept

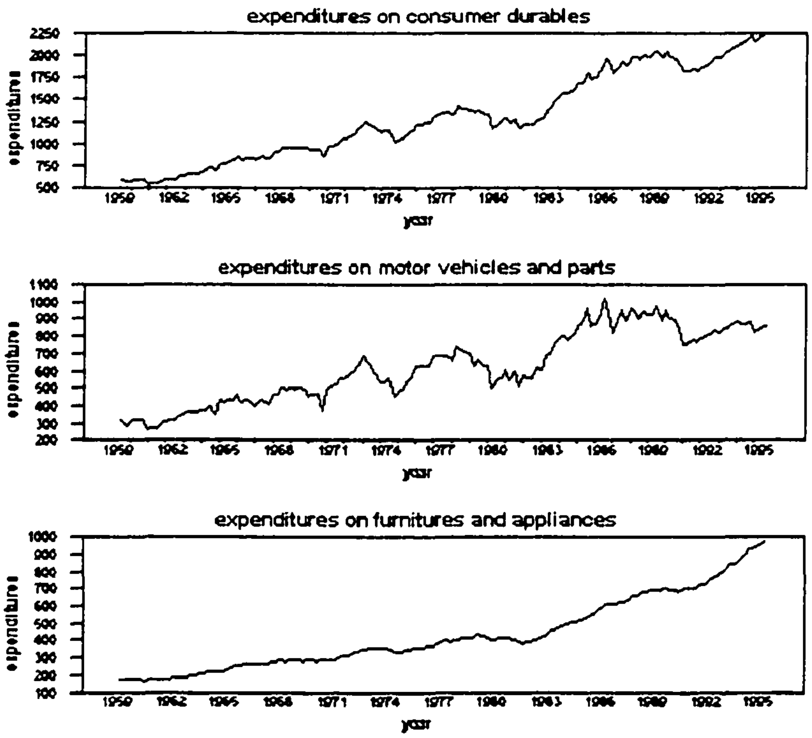

Figure 5.1. Seasonally adjusted durable goods expenditure series

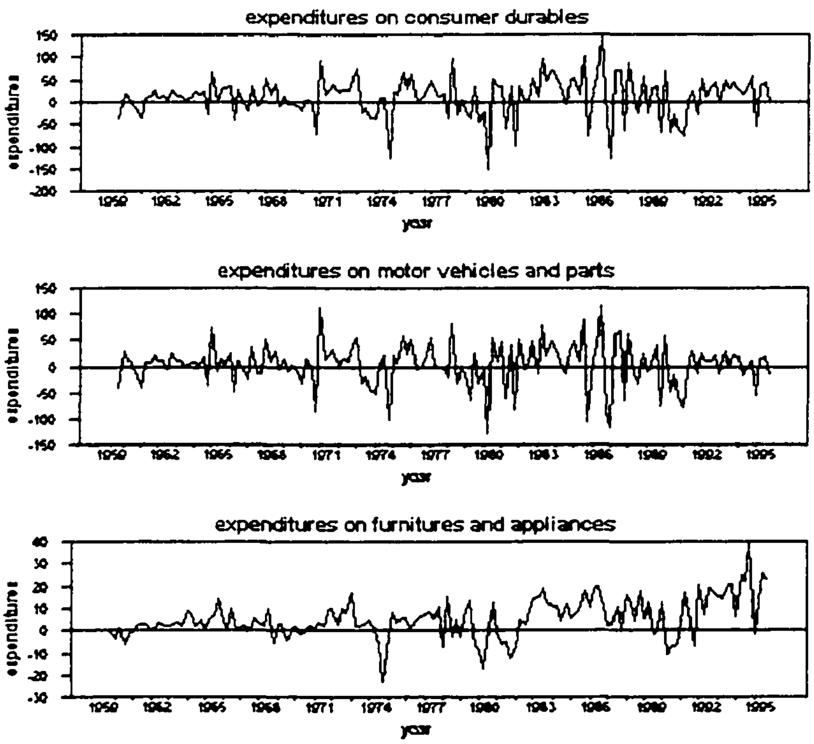

Figure 5.2. First differenced seasonally adjusted expenditures series

Table 5.1. Results of unit root tests (ADF tests)

Series	Observations	Lags	Statistics
Consumer durables	144	4	$\tau_\tau = -2.96$
I (1)			$\phi_3 = 4.51$
Motor vehicle/parts	144	4	$\tau_\tau = -2.75$
I (1)			$\phi_3 = 3.87$
Furniture/Appliances	144	1	$\tau_\tau = 1.40$
I (1)			$\phi_3 = 7.33^{*}$
Furniture/Appliances	144	1	$\tau_\tau = 4.32^{**}$
I (2)			$\phi_3 = 9.47^{**}$

Note: *significant at 5% level
**significant at 1% level
I (1): test of first order integration
I (2): test of second order integration

and the time trend term is used to test for presence of unit root in the seasonally adjusted consumer durable expenditures series and its subcategories. Initially the equation is estimated with autoregressive order of lag 4, following the restriction of Said and Dickey (1984). Then the standard t-test and F-test are used to pare down the lags in the equation. If none of the lags are significant, then the Schwartz Bayesian criterion is used to determine the lag length. For the consumer durables expenditures series and the motor vehicle series, the sample estimates of the statistics τ_τ are less than the critical value at the 5 % significance level. Thus, the null hypothesis that the series has a unit root cannot be rejected for either of the series. The next step is to test for the appropriateness of including the deterministic time trend term with the ϕ_3–statistics. Since the sample estimates of the ϕ_3–statistics for the consumer durables series and the motor vehicle series are less than the critical value at the 5 % significance level, the joint hypothesis of a unit root and no time trend in the first difference cannot be rejected for either of the two series. For the furnitures and household appliances series, however, the joint hypothesis is rejected at 5% significance level, which suggests that the series either has a deterministic time trend or two unit roots. The ADF test is performed on second difference of the sequence to test whether the sequence is second order integrated; I (2). The sample estimates of the τ_τ-statistic is significant at 1 % significance level, thus, rejecting the null hypothesis that the series is I (2). The joint hypothesis of a unit root and no time trend in the second difference is also rejected for the furnitures and household appliances series at conventional significance levels, which suggests that including a deterministic time trend is appropriate for the series. Hence it is concluded that the furnitures and household appliances series is first order integrated with a deterministic time trend.

Since the models are specified in first difference, the furnitures and household appliances series is detrended by a quadratic time trend before first differencing to yield a stationary series, whereas the first differenced sequences of the consumer durables and the motor vehicles and parts series yield stationary series without deterministically detrending. We were unable to find any specific reason for the presence of a deterministic trend in the furnitures/appliances series that are absent in other two series. The procedure to detrend a sequence with a deterministic time trend prior to first differencing, however, were used by Caballero (1990). The sample autocorrelation for the first differenced series of consumer durables expenditures and expenditures on the two subcategories are displayed in Figure 5.3. The dotted lines display the two standard deviation band (95% confidence interval). For the seasonally adjusted consumer durables expenditures series, the change in durable expenditures sequence exhibits a series close to a white noise process. However, the series displays

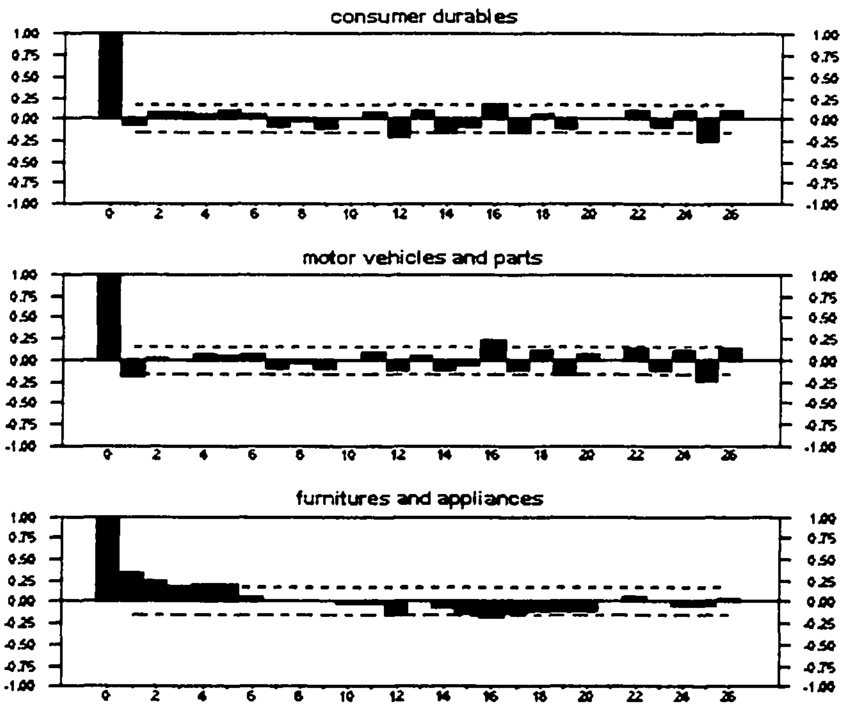

Figure 5.3. Sample autocorrelation of first differenced durable good series

significant negative spikes between lag 12 and lag 17, and at lag 25. The seasonally adjusted motor vehicles and parts series exhibit similar autocorrelation patterns as the consumer durables expenditures series except that a significant negative spike is displayed at lag 1. The autocorrelations of the furniture and household appliances series also exhibit significant spikes clustered around lag16. A decaying pattern of the autocorrelation function is suggestive of an AR (1) process.

Table 5.2 presents the Box-Jenkins ARMA model estimation results of the seasonally adjusted consumer durables expenditures series. Estimation of the MA (1) model shows that the MA coefficient is insignificant and close to zero, which is consistent with previous study results that the change in durable expenditures exhibits a series close to a white noise process. The Ljung-Box Q-statistics of the residuals indicates that the parsimonious MA (1) model does not capture the long-term dynamics of the consumer durables expenditures series. The Ljung-Box Q-statistics at 20 lags and 30 lags indicates the significance of the longer lags. Estimation of the MA (||12,17, 25||) process shows that the Q-statistics at 20 lags and 30 lags do not indicate any significant autocorrelations in the residuals. The result suggests that an MA process incorporating significant longer lags does better job of capturing the long-run dynamics. In terms of the models presented in this paper, significant coefficients of MA (||12||) and MA (||25||) could reflect the longer average durable goods purchase interval. Since different durable goods categories would have different purchase interval, examining the two different subcategories, motor vehicles series and furniture/appliances series, would be more conducive to model identification.

Table 5.2. Estimates of ARMA models of consumer durables series

MA (1): $\Delta C_t = 11.62 + \varepsilon_t - 0.080\,\varepsilon_{t-1}$
(3.42) (-0.96)

AIC = 1812, SBC = 1818,
Q (10) = 9.82 (0.28), Q (20) = 38.18 (0.00), Q (30) = 60.26 (0.00)

MA (||12||): $\Delta C_t = 11.55 + \varepsilon_t - 0.199\,\varepsilon_{t-12}$
(3.87) (-2.39)

AIC = 1806, SBC = 1812,
Q (10) = 9.14 (0.33), Q (20) = 31.05 (0.03), Q (30) = 53.84 (0.00)

MA (||25||): $\Delta C_t = 11.58 + \varepsilon_t - 0.422\,\varepsilon_{t-25}$
(5.03) (-4.98)

AIC = 1796, SBC = 1802,
Q (10) = 9.22 (0.32), Q (20) = 33.62 (0.01), Q (30) = 47.32 (0.01)

MA (||12, 17, 25||): $\Delta C_t = 11.38 + \varepsilon_t - 0.128\varepsilon_{t-12} - 0.179\varepsilon_{t-17} - 0.421\varepsilon_{t-25}$
(8.11) (-1.57) (-2.16) (-4.83)
AIC = 1794, SBC = 1806,
Q (10) = 7.66 (0.26), Q (20) = 19.81 (0.23), Q (30) = 30.59 (0.24)

note: 1.sample period:1960:1 – 1995:4
2. ( ): t –statistics
3. AIC= Akaike information criterion
SBC= Schwartz Bayesian Criterion
Q ( ) = Box-Ljung Q-statistics for residuals (significance level in the parentheses)

Table 5.3 presents the Box-Jenkins ARMA model estimation results of the seasonally adjusted motor vehicle and parts series. Estimated coefficients of the MA (1), AR (1), MA (||16||) and MA (||25||) models are shown to be significant. The MA (||25||) model performs better than other models based on the goodness of fit measures such as the Akaike information criterion and the Schwartz Bayesian Criterion. Although the MA (||1, 25||) model performs equally well as the MA (||25||) model, the first order coefficient turns out to be insignificant. Thus, the MA (||25||) model is selected among the candidate models. The MA (||25||) model is consistent with the time-series implication of the base model when the purchase interval is 25 quarters. The base model predicts MA (||25||) process with coefficient around –0.26 when the purchase interval is 25 quarters (or 72 months). Significant MA (||25||) coefficient implies that the average purchase interval for automobiles is around 6 years, which seems intuitively more plausible than an average purchase interval of 3 months implied by a MA (1) model. However, one should be cautious about interpreting the estimation result because the estimation of longer lags is subject to a small sample bias.

Figure 5.4 displays autocorrelation functions of the residuals of the MA (||25||) model. The autocorrelations are less than two standard deviation from zero except for the autocorrelation at lag 16, which indicates that the model fits the data quite well. As an additional diagnostic check for the model accuracy, the structural change of the model coefficients is tested by splitting the sample into two sub-periods. Due to the higher volatility since the late 1970's, the sample is split into two sub-periods 1960:1 to 1978:4 and 1979:1 to 1995:4. Null hypothesis of no structural change in the coefficients is tested using the F-test.

Table 5.3. Estimates of ARMA models of motor vehicle series

MA (1): $\Delta C_t = 3.88 + \varepsilon_t - 0.210\,\varepsilon_{t-1}$
(1.46) (-2.54)

AIC = 1783, SBC = 1789,
Q (10) = 9.41 (0.31), Q (20) = 32.17 (0.02), Q (30) = 51.63 (0.00)

AR (1): $\Delta C_t = 3.86 - 0.215\Delta C_t + \varepsilon_t$
(1.39) (-2.64)

AIC = 1782, SBC = 1788,
Q (10) = 9.47 (0.30), Q (20) = 31.44 (0.03), Q (30) = 50.18 (0.01)

MA (||16||): $\Delta C_t = 4.03 + \varepsilon_t + 0.217\varepsilon_{t-16}$
(1.01) (2.61)

AIC = 1781, SBC = 1787,
Q (10) = 9.22 (0.32), Q (20) = 26.37 (0.09), Q (30) = 49.58 (0.00)

MA (||25||): $\Delta C_t = 4.18 + \varepsilon_t - 0.374\,\varepsilon_{t-25}$
(1.84) (-4.33)

AIC = 1775, SBC = 1781,
Q (10) = 8.72 (0.37), Q (20) = 32.96 (0.02), Q (30) = 46.85 (0.01)

MA (||1, 25||): $\Delta C_t = 4.15 + \varepsilon_t - 0.118\varepsilon_{t-1} - 0.322\varepsilon_{t-25}$
(2.03) (-1.35) (-3.69)

AIC = 1775, SBC = 1784,
Q (10) = 7.61 (0.37), Q (20) = 29.23 (0.03), Q (30) = 42.55 (0.03)

note: 1.sample period:1960:1 – 1995:4
2. ( ): t –statistics
3. AIC= Akaike information criterion
SBC= Schwartz Bayesian Criterion
Q ( ) = Box-Ljung Q-statistics for residuals (significance level in the parentheses)

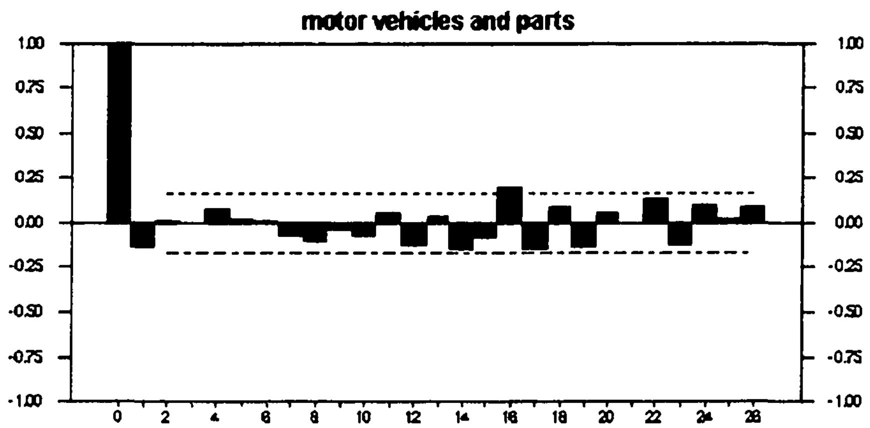

Figure 5.4. ACF of the residuals of MA (||25||) model

The estimates of the MA (||25||) model for the two sub-periods and the structural F-test result is presented in Table 5.4. The F-test result indicates that we cannot reject the null hypothesis of no structural change in the coefficients at 5% significance level. Thus we can conclude that there is no evidence of structural change in the coefficients of the selected MA (||25||) model.

Table 5.4. Structural change test of the MA (||25||) model

(1960:1 ~ 1978:4): $\Delta C_t = 5.57 + \varepsilon_t - 0.32\ \varepsilon_{t-25}$

(1979:1 ~ 1995:4): $\Delta C_t = 2.47 + \varepsilon_t - 0.45\ \varepsilon_{t-25}$

$SSR = 219490$, $SSR_1 = 77635$, $SSR_2 = 141090$.

$F(2,140) = 0.245$, $F_{0.05,2,100} = 3.09$.

Table 5.5 presents the Box-Jenkins ARMA model estimation results of the seasonally adjusted furnitures and household appliances series. Estimation of the furnitures and household appliances series shows that the MA (1) coefficient is significantly positive rejecting the base model implication. Estimation of the AR (1) model, however, shows that the model fits quite well with a positive AR coefficient. Significant AR coefficient could be due to the habit persistence effect discussed in section 3.2. According to the AR (1) model, the AR coefficient of 0.34 reflects the degree of habit persistence that carries over from the previous durable purchase. Estimation of the ARMA models resulted in positive AR coefficients and negative MA coefficients. The ARMA (1, ||12||) model performs better than other models based on the goodness of fit measures and the diagnostic criterion of Ljung-Box Q-statistics. The ARMA (1, ||12||) model is consistent with the time-series implication of the habit persistence model when the purchase interval is 36 month. The habit persistence model predicts an ARMA (1, ||12||) process when the purchase interval is 12 quarters with positive AR coefficient reflecting the degree of habit persistence and negative MA coefficient. The estimation result implies that an average purchase interval for furnitures and household appliances is around 3 years. As in the MA (||25||) model of the motor vehicle series, further diagnostic checks are performed with the ARMA (1, ||12||) model by plotting the residual correlogram and testing the structural change of the coefficients. Figure 5.5 displays the autocorrelation function of the residuals of the ARMA (1, ||12||) model. The autocorrelations are less than the two standard deviation from zero, which indicates that the model fits the data quite well. The estimates of the ARMA (1, ||12||) model for the two sub-periods and the structural F-test result are presented in Table 5.6. The F-test result indicates that we cannot

Table 5.5. Estimates of ARMA models of furnitures/appliances series

MA (1): $\Delta C_t = 0.85 + \varepsilon_t + 0.26\varepsilon_{t-1}$
(1.06) (3.14)

AIC = 1304, SBC = 1310,
Q (10) = 17.16 (0.03), Q (20) = 35.66 (0.01), Q (30) = 38.43 (0.09)

AR (1): $\Delta C_t = 0.89 + 0.34\Delta C_{t-1} + \varepsilon_t$
(0.94) (4.26)

AIC = 1300, SBC = 1306,
Q (10) = 9.37 (0.31), Q (20) = 23.99 (0.16), Q (30) = 27.51 (0.49)

ARMA (1,1): $\Delta C_t = 1.13 + 0.80\Delta C_{t-1} + \varepsilon_t - 0.54\,\varepsilon_{t-1}$
(0.78) (6.78) (-3.36)

AIC = 1296, SBC = 1305,
Q (10) = 3.88 (0.79), Q (20) = 15.10 (0.59), Q (30) = 21.29 (0.77)

ARMA (1, ||12||): $\Delta C_t = 0.68 + 0.36\Delta C_{t-1} + \varepsilon_t - 0.30\varepsilon_{t-12}$
(0.98) (4.47) (-3.26)

AIC = 1293, SBC = 1302,
Q (10) = 7.96 (0.34), Q (20) = 16.08 (0.52), Q (30) = 18.54 (0.89)

ARMA (1, ||16||): $\Delta C_t = 0.74 + 0.30\Delta C_{t-1} + \varepsilon_t - 0.15\,\varepsilon_{t-16}$
(0.95) (3.72) (-1.57)

AIC = 1299, SBC = 1308,
Q (10) = 8.68 (0.28), Q (20) = 22.66 (0.16), Q (30) = 27.36 (0.44)

note: 1.sample period:1960:1 – 1995:4
2. ( ): t –statistics
3. AIC= Akaike information criterion
SBC= Schwartz Bayesian Criterion
Q ( ) = Box-Ljung Q-statistics for residuals (significance level in the parentheses)

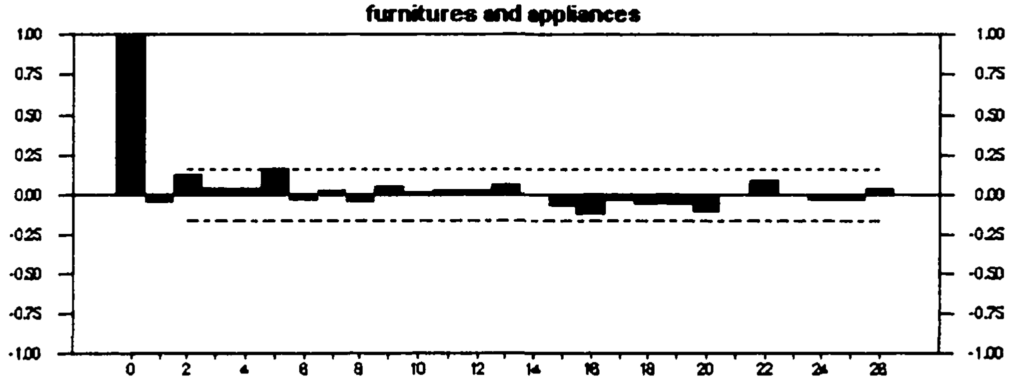

Figure 5.5. ACF of the residuals of ARMA (1, ||12||) model

Table 5.6. Structural change test of the ARMA (1, ||12||) model

(1960:1 ~ 1978:4): $\Delta C_t = 0.56 + 0.19\Delta C_{t-1} + \varepsilon_t - 0.24\ \varepsilon_{t-12}$

(1979:1 ~ 1995:4): $\Delta C_t = 1.27 + 0.41\Delta C_{t-1} + \varepsilon_t - 0.29\ \varepsilon_{t-12}$

$SSR = 7640$, $SSR_1 = 2118$, $SSR_2 = 5464$.

$F(3,138) = 0.348$, $F_{0.05,3,100} = 3.14$.

reject the null hypothesis of no structural change in the coefficients at 5% significance level. Thus we can conclude that there is no evidence of structural change in the coefficients of the selected ARMA (1, ||12||) model.

The estimation results using the two subcategories of the durable goods expenditures show that the representative-agent rational expectations- permanent income model is capable of explaining the quarterly aggregated seasonally adjusted consumer durable goods expenditures once the infrequent microeconomic action is incorporated into the model and the time aggregation is explicitly taken into account. Estimation results imply that the average purchase interval of automobiles is 25 quarters and that of furnitures and household appliances is 12 quarters. The implications seem intuitively plausible. The structural tests indicate that the model coefficients for the MA (||25||) and ARMA (1, ||12||) are structurally stable over time, which gives additional support for the accuracy of the models.

A potential problem of estimating the longer lags, however, is the possibility of the small sample bias given the limited number of observations. Thus one should take caution when interpreting the estimation results. One possible explanation for the significance of the habit persistence effect in the furnitures and household appliances series which is absent in the motor vehicle and parts series is that the furnitures and electronic appliances are less expensive than automobiles, allowing consumers to upgrade these items more easily. Upgrading household appliances or furnitures is facilitated by the extensive use of one-time payment through credit cards. Automobile purchases, on the other hand, are more susceptible to liquidity constraints and thus more difficult to upgrade since many consumers still depend on bank financing to purchase cars.

Another potential problem of estimating the longer lags with the multi-period purchase interval framework is the possibility that the significant longer lags could be due to the seasonality rather than reflecting longer purchase intervals of durable goods. Estimations using the deseasonalized data do not guarantee a complete removal of seasonal variations. Therefore in order to investigate the issue, a seasonal unit root test is performed on the seasonally adjusted durable expenditures series and the two subcategories. Table 5.7 presents the result of the seasonal unit root tests. The lag lengths are determined using the same procedure as the unit root tests of the seasonally adjusted durables series.

Table 5.7. Results of seasonal unit root tests of deseasonalized data.

Series	Observations	lags	Statistics
Consumer Durables Expenditures	144	4	$t(\gamma_1 = 0) = -0.38$ $t(\gamma_2 = 0) = 3.71^{**}$ $F(\gamma_5 = \gamma_6 = 0) = 22.12^{**}$
Motor vehicle And parts	144	4	$t(\gamma_1 = 0) = -1.71$ $t(\gamma_2 = 0) = 3.51^{**}$ $F(\gamma_5 = \gamma_6 = 0) = 21.39^{**}$
Furnitures And Appliances	144	4	$t(\gamma_1 = 0) = 2.26$ $t(\gamma_2 = 0) = 4.57^{**}$ $F(\gamma_5 = \gamma_6 = 0) = 17.21^{**}$

Note: * significant at 5%
** significant at 1%

For the consumer durables expenditures series and the two subcategories, the sample estimates of the first t-statistics; $t(\gamma_1 = 0)$, are lower than the critical value at the 5 % significance level. Hence, the presence of nonseasonal unit root cannot be rejected for these series. The presence of the semi-annual unit root and annual unit root, on the other hand, are rejected at 1% significant level for the three series. The seasonal unit root test results suggests that seasonal variations at semi-annual and annual frequencies are effectively removed with the deseasonalized data. Hence, the possibility that the significant longer lags could be due to the remaining seasonality is minimal.

5.2. Estimation with seasonally unadjusted series

Figure 5.6 displays the seasonally unadjusted series on consumer durables expenditures and subcategories. The consumer durables series reveals an upward sloping curve with seasonal variation. The furnitures and household appliances series displays a distinct seasonal variation that appears to be constant over time, whereas the seasonal fluctuation of the motor vehicle and parts series appear to be less pronounced. Figure 5.7 displays the first-difference series of the seasonally unadjusted durable expenditures and its subcategories. The trend has been effectively removed by the first differencing of the series.

In the first part of the analysis, the model including the deterministic seasonal dummies is estimated to examine the percentage of non-trend variation explained by seasonal variation. The first-differenced series are estimated to remove the trend. The estimated results are presented in Table 5.8. It reports the seasonal patterns of the change in durable goods expenditures and its two subcategories. All three series exhibit seasonal decrease in first and third quarter and seasonal increase in the fourth quarter. The fourth period increase is probably due to the Christmas season effect. The R^2 of the durable goods series implies that 89% of non-trend variation is explained by the seasonal variation. For the subcategories, the

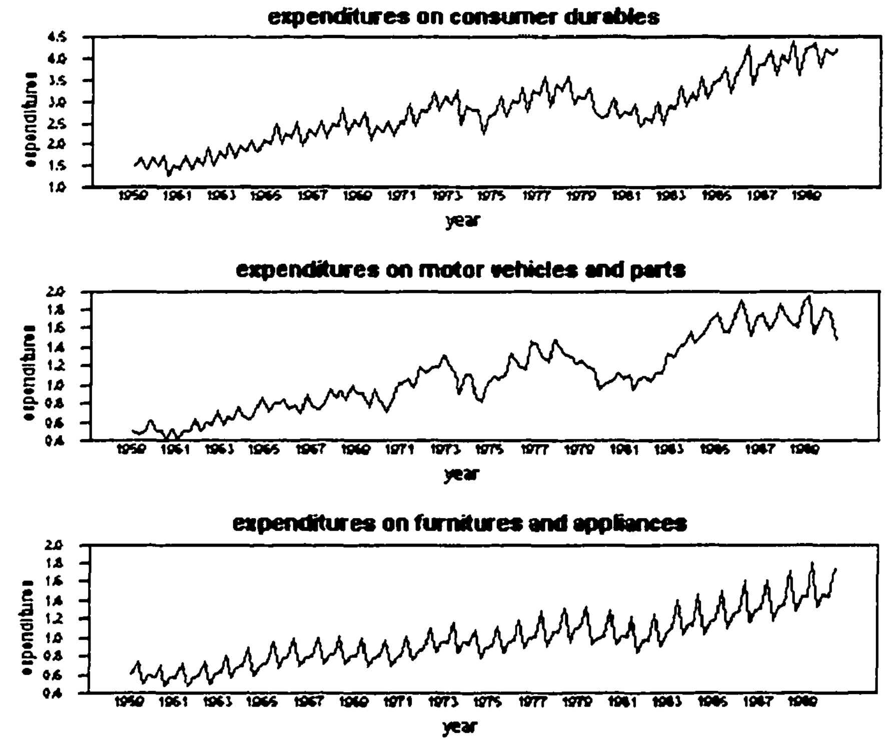

Figure 5.6. Seasonally unadjusted durable goods expenditures series

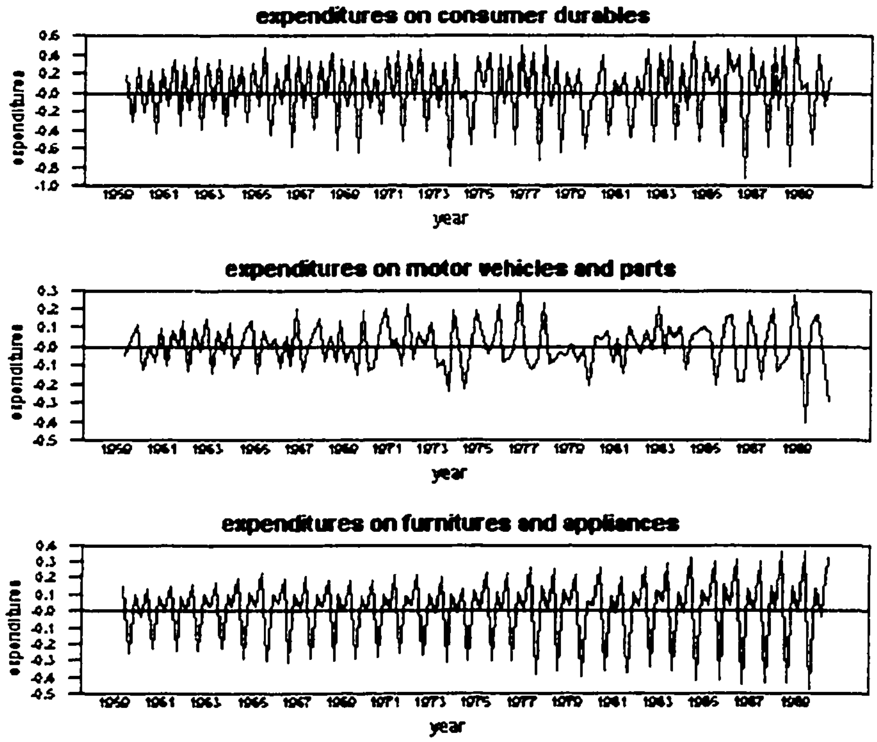

Figure 5.7. First differenced seasonally unadjusted expenditures series

Table 5.8. Seasonal patterns in detrended durable goods expenditures series

series	quarter 1	quarter 2	quarter 3	quarter 4	R^2
durables	-0.415	0.009	-0.861	0.338	0.89
autos	-0.186	-0.185	-0.147	0.138	0.37
furnitures	-0.092	0.126	-0.431	0.107	0.94

percentages are 37% and 94% respectively. The seasonal variation seems to dominate the overall variation in quarterly durable expenditures series and the furnitures and household appliances series but less so with the motor vehicle series. These results are consistent with the graphical analysis and suggest that seasonal fluctuation in quarterly durable goods expenditures are driven by the seasonal fluctuation of the furnitures and appliances series.

The next step is to further test for the presence of stochastic seasonality. The procedure developed by Hylleberg et al. (1990) is followed to test for the presence of seasonal unit roots in the consumer durable goods expenditures series and its two subcategories. The results of seasonal unit root tests are presented in Table 5.9. The lag lengths are determined using the same procedure as the unit root tests of the seasonally adjusted durables series. For the consumer durables expenditures series and the furnitures and household appliances series, the sample estimates of all three statistics are lower than the critical value at the 5 % significance level. Hence, the presence of seasonal unit roots at all three frequencies cannot be rejected for either of these two series. For the motor vehicle and parts series, the presence of a nonseasonal unit root cannot be rejected at the 5 % significance level, while the presence of the semi-annual unit root and annual unit root are not rejected at 1% significant level. The

Table 5.9. Results of seasonal unit root tests.

Series	Observations	lags	Statistics
Consumer Durables Expenditures	124	4	$t(\gamma_1 = 0) = -0.99$ $t(\gamma_2 = 0) = 0.41$ $F(\gamma_5 = \gamma_6 = 0) = 1.28$
Motor vehicle And parts	124	1	$t(\gamma_1 = 0) = -1.22$ $t(\gamma_2 = 0) = 2.56^*$ $F(\gamma_5 = \gamma_6 = 0) = 4.94^*$
Furnitures And Appliances	124	1	$t(\gamma_1 = 0) = -0.90$ $t(\gamma_2 = 0) = -1.25$ $F(\gamma_5 = \gamma_6 = 0) = 0.18$

Note: * significance at 5% but not significant at 1%

seasonal unit root test results suggests that seasonal differencing filter could be applied to the durable goods series and the two subcategories. Based on these results, the seasonally unadjusted series of the durable expenditures series and the two subcategories are estimated using the stochastic seasonal model.

The stochastic seasonal model (equation 3.13) assumes that seasonal unit roots are present. Seasonally differencing a series using the seasonal difference filter is equivalent to canceling out $(1 - L^d)$ term from both sides of the equation;

$$(1 - L^d)(1-L)C_t = (1 - (1 - \delta)^d L^d)(1 - L^d)\varepsilon_t$$

which gives the following residual model:

$$(1-L)C_t = (1 - (1 - \delta)^d L^d)\varepsilon_t$$

Using the quarterly aggregated durable expenditures series, the period d equals 4, and the residual model predicts a MA (|| 4 ||) process. The consumer durables and two subcategories of the seasonally unadjusted series are seasonally differenced by the seasonal differencing filter to yield a residual series. The residual series is then estimated by a Box-Jenkins method. The above procedure used to estimate the seasonally unadjusted series is essentially equivalent to estimating the Seasonal ARIMA (SARIMA) models. Suppose it is possible to find a linear transformation of the series$\{C_t\}$ that yields a stationary series $\{D_t\}$ after seasonal differencing. Thus, $(1\text{-}L^4)C_t = D_t$. The next step is to examine the sample autocorrelation and to model the $\{D_t\}$ series. The sample autocorrelation (correlogram) for the three series are displayed in Figure 5.8. The dotted lines represent the two standard deviation band. For the seasonally unadjusted durable expenditures series, the motor vehicle series and the furnitures and appliances series, the autocorrelation of the seasonally differenced / first differenced series display significant spike at lag 4 which suggests a MA (|| 4 ||) process. The consumer durables series and the motor vehicle series also display significant spikes at lags 1 and 11, which is suggestive of an MA (1) or a MA (||11||) process.

Table 5.10 presents the Box-Jenkins ARMA model estimation results. For the consumer durables series, estimation of the MA (1) model resulted in significant correlation among the residuals. The Ljung-Box Q-statistics are significant at 5% level for 10 lags, 20 lags, and 30 lags, which suggests that the model is not capturing the movement in the durables expenditures sequence. Estimation of MA (|| 4 ||) model, on the other hand, gives highly significant MA coefficient with the Ljung-Box Q-statistics indicating no significant residual correlation. Thus the MA (|| 4 ||) model appear to fit the data well. The MA (|| 4 ||) model also performs well for the motor vehicle series and the furnitures/appliances series. The MA (|| 4 ||) process appears to capture the short-run and the long-run dynamics of the furnitures/appliances series quite well. The Ljung-Box Q-statistics indicates little correlation among the residuals. For the motor vehicle series, the MA (||1, 4 ||) model performs better

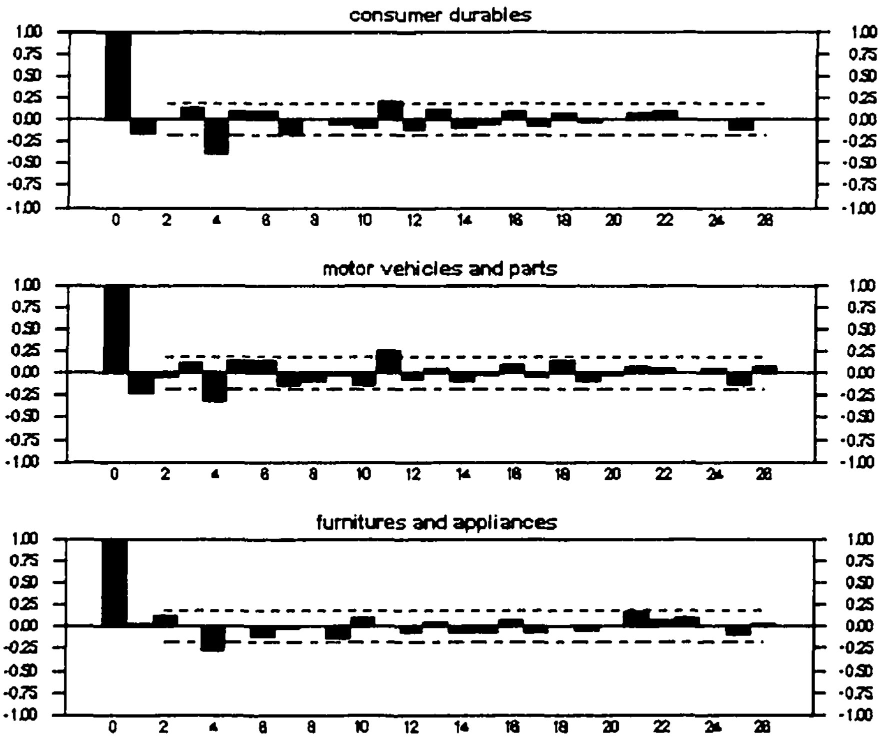

Figure 5.8. Sample autocorrelation of seasonal differenced/first differenced series

Table 5.10.Estimates of ARMA models of seasonally unadjusted series

Seasonally unadjusted durable consumption series

MA (1): $\Delta C_t = -0.001 + \varepsilon_t - 0.172\, \varepsilon_{t-1}$
(-0.07) (-1.897)

AIC = 118, SBC = 123,
Q (10) = 26.11 (0.00), Q (20) = 36.97 (0.01), Q (30) = 45.73 (0.02)

MA (|| 4 ||): $\Delta C_t = -0.0001 + \varepsilon_t - 0.74\, \varepsilon_{t-4}$
(-0.01) (-11.15)

AIC = 81, SBC = 87,
Q (10) = 7.99 (0.43), Q (20) = 17.29 (0.50), Q (30) = 23.75 (0.69)

Seasonally unadjusted motor vehicles and parts series

MA (1): $\Delta C_t = -0.0001 + \varepsilon_t - 0.28\, \varepsilon_{t-1}$
(-0.12) (-3.17)

AIC = 40, SBC = 46,
Q (10) = 26.29 (0.00), Q (20) = 38.73 (0.00), Q (30) = 46.09 (0.02)

MA (|| 4 ||): $\Delta C_t = -0.0006 + \varepsilon_t - 0.68\, \varepsilon_{t-4}$
(-0.21) (-9.32)

AIC = 18, SBC = 24,
Q (10) = 13.95 (0.08), Q (20) = 27.85 (0.06), Q (30) = 37.04 (0.12)

note: 1.sample period:1961:1 – 1995:4
2. ( ): t –statistics
3. AIC= Akaike information criterion
SBC= Schwartz Bayesian Criterion
Q ( ) = Box-Ljung Q-statistics for residuals (significance level in the parentheses)

Table 5.10. (continued)

MA(|| 11 ||): $\Delta C_t = -0.001 + \varepsilon_t + 0.26\, \varepsilon_{t-11}$
(-0.09) (2.78)

AIC = 40, SBC = 46,
Q (10) = 32.17 (0.00), Q (20) = 39.59 (0.00), Q (30) = 48.37 (0.01)

MA (||1, 4||): $\Delta C_t = -0.001 + \varepsilon_t - 0.53\varepsilon_{t-1} - 0.70\, \varepsilon_{t-4}$
(-0.63) (-7.91) (-10.23)

AIC = 14, SBC = 22,
Q (10) = 43.95 (0.00), Q (20) = 56.28 (0.00), Q (30) = 76.47 (0.00)

Seasonally unadjusted furnitures and household appliances series

MA (|| 4 ||): $\Delta C_t = 0.0003 + \varepsilon_t - 0.35\, \varepsilon_{t-4}$
(0.189) (-3.89)

AIC = -292, SBC = -287,
Q (10) = 8.47 (0.39), Q (20) = 13.63 (0.75), Q (30) = 20.77 (0.83)

than other models based on goodness of fit measures. The MA (||1, 4||) model, however, suffers from significant residual autocorrelation that is absent in the MA (|| 4 ||) model. Estimation of the MA (|| 4 ||) model for the durable consumption series and motor vehicle series resulted in a significant coefficient of –0.74 and –0.68 respectively. Estimation of the furnitures and household appliance series, however, gives a slightly lower MA coefficient of around -0.35. The estimation results of seasonally unadjusted series are consistent with the time-series implication of the stochastic seasonal model, which predicts MA (|| 4 ||) process with MA coefficient around 0.8. These estimation results suggest that seasonal variation explain a substantial portion of the total variation in consumer durable goods expenditures. As in the estimations of the seasonally adjusted series, further diagnostic check of plotting the residual correlogram and testing the structural change are performed with the MA (|| 4 ||) models. Figure 5.9 displays the autocorrelation function of the residuals. The residual autocorrelations are all within the two standard deviation from zero, which indicates that the models fit the data well. The sample is split into half to test for the structural change of coefficients. The estimates of the MA (|| 4 ||) model for the two sub-periods and the structural F-test result are presented in Table 5.11. The F-test result indicates that we cannot reject the null hypothesis of no structural change in the coefficients at 5% significance level for all three series. Thus we can conclude that there is no evidence of structural change in the coefficients of the MA (|| 4 ||) model for any of the three series.

Estimation of the stochastic seasonal model, variation of the base model, showed that the MA (|| 4 ||) model is capable of mimicking the seasonally unadjusted consumer durable expenditures series and the two subcategory series with substantial accuracy. Therefore, the representative-agent rational expectations-permanent income model is capable of explaining the dynamics of the seasonally unadjusted consumer durable goods expenditures series. These results suggest that it may be useful to model seasonality explicitly rather than removing it using ad hoc seasonal adjustment. The procedure underlying seasonal adjustment assumes that a seasonal and nonseasonal fluctuations are independent and could be decomposed into seasonal and nonseasonal components. If seasonal, trend and cyclical components are difficult to separate, the seasonal adjustment may remove valuable information from an economic time-series and the usefulness of the seasonally adjusted data is questionable in this case. It is quite

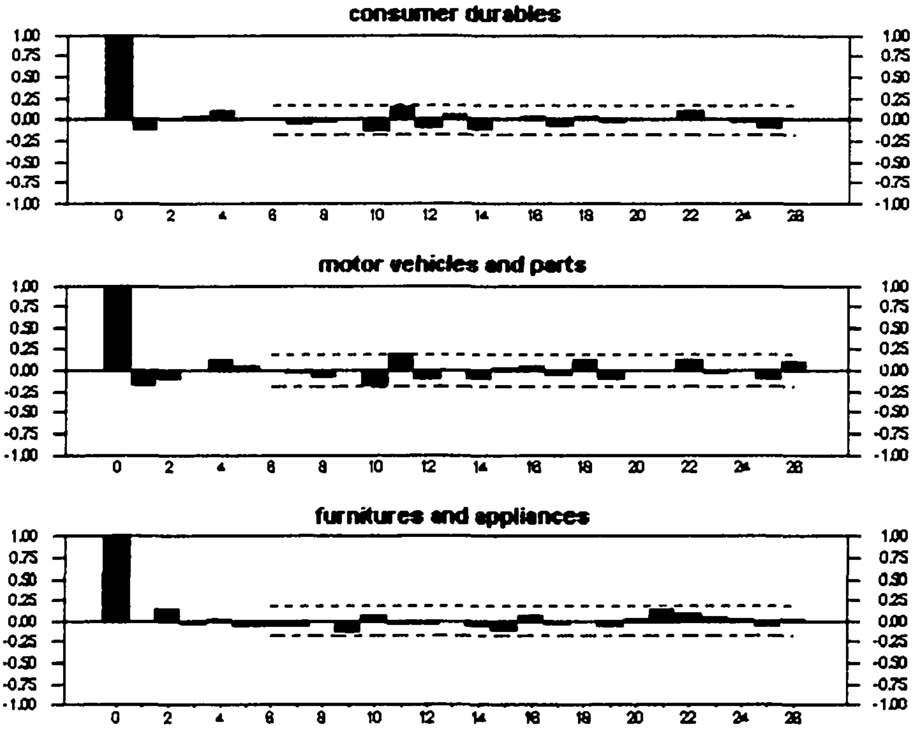

Figure 5.9. ACF of the residuals of the MA (|| 4 ||) model

Table 5.11. Structural change test of the MA (|| 4 ||) models

Seasonally unadjusted durable consumption series

(1961:1 ~ 1975:4): $\Delta C_t = 0.0003 + \varepsilon_t - 0.72\ \varepsilon_{t-4}$

(1976:1~ 1995:4): $\Delta C_t = -0.004 + \varepsilon_t - 0.70\ \varepsilon_{t-4}$

$SSR = 1.960$, $SSR_1 = 0.675$, $SSR_2 = 1.258$.

$F(2,116) = 0.838$, $F_{0.05,2,100} = 3.09$.

Seasonally unadjusted motor vehicle series

(1961:1 ~ 1975:4): $\Delta C_t = -0.0003 + \varepsilon_t - 0.76\ \varepsilon_{t-4}$

(1976:1~ 1995:4): $\Delta C_t = -0.004 + \varepsilon_t - 0.50\ \varepsilon_{t-4}$

$SSR = 1.128$, $SSR_1 = 0.337$, $SSR_2 = 0.786$.

$F(2,116) = 0.266$, $F_{0.05,2,100} = 3.09$.

Seasonally unadjusted furnitures/appliances series

(1961:1 ~ 1975:4): $\Delta C_t = 0.0009 + \varepsilon_t - 0.42\ \varepsilon_{t-4}$

(1976:1~ 1995:4): $\Delta C_t = -0.0009 + \varepsilon_t - 0.36\ \varepsilon_{t-4}$

$SSR = 0.085$, $SSR_1 = 0.032$, $SSR_2 = 0.051$.

$F(2,116) = 1.173$, $F_{0.05,2,100} = 3.09$.

plausible to include seasonal fluctuation in RE-PIH models because seasonal fluctuations in consumption are predictable and it is likely that consumers anticipate them and adjust their consumption behavior accordingly. Yet most economic analyses on RE-PIH have been carried out with seasonally adjusted or annual data and therefore include a bias toward rejecting the RE-PIH model.

Miron (1986) and others have pointed out that previous studies on consumption may have reached a biased conclusion about the business cycle fluctuations in consumption due to the exclusion of seasonal fluctuations. They showed that seasonal variation accounted for a significant portion of overall short term variation in durable expenditures series. More recently, researchers such as Canova and Ghysels(1994) have investigated the issue of independence of seasonal, trend and cyclical fluctuations and showed that seasonal and cyclical variations interact in an important way. They concluded that one cannot understand the business cycle without also understanding the seasonal cycle. The estimation results in this paper supports the findings from these recent studies on the business cycle, in that incorporating seasonality directly in the model explains the aggregate dynamics of an intuitively plausible RE-PIH.

6. CONCLUSION

The rational expectations-permanent income hypothesis assumes that rational agents use expected future income and current wealth to determine an optimal consumption path. Hall (1978) formulated the time-series representation of the RE-PIH and showed that the expected marginal utility is a martingale process. Mankiw (1982) applied Hall's model to durable goods consumption and derived the result that the change in consumption expenditures on durable goods should follow a first order moving average process with the coefficient approximating –0.95 for quarterly data. The empirical test of the joint hypothesis, however, has been rejected using the seasonally adjusted U.S. quarterly data, questioning the validity of the joint hypothesis.

Caballero (1990) argued that the parsimonious MA (1) model is not likely to detect the spread out consumer responses to aggregate income innovations and that a nonparsimonious MA (q) process is necessary to capture the persistence of aggregate disturbances due to the disperse response. He showed that the sum of MA (q) coefficients from estimating changes in annual consumer durables expenditures approximates the magnitude predicted by the theory and thus concluded that the representative agent based RE-PIH model is a useful way to think about the long-run response of durables expenditures to the aggregate income shocks. In the subsequent paper, Caballero (1993) argued that the slow adjustment could reflect intermittent adjustment by consumers because of the adjustment cost involved in purchasing durable goods. To incorporate such realistic microeconomic feature into a model, Caballero abandoned the rational expectations optimization framework and introduced a framework based on (S, s) inventory rule where the dynamic analysis of stochastically heterogeneous units is made operational.

This paper presented a permanent income model based on the rational expectations optimization framework that is capable of explaining the quarterly aggregate durables expenditures series. A novel feature of the analysis presented in this paper is that the observed infrequent purchases of durable goods by the consumers are incorporated into the rational expectations optimization model and that the time aggregation problem is explicitly addressed to investigate the aggregate dynamics of the durables expenditures. The time-series implication of the base model is that the change in durables expenditures is a function of the durable goods purchase interval. The seasonal model, the variation of the base model, implies that the change in durables expenditures should follow an MA (|| 4 ||) process after removing the seasonal differencing filter.

Estimation results show that the representative-agent rational expectations-permanent income model is capable of explaining the aggregate dynamics of the consumer durable goods expenditures once the infrequent microeconomic action is incorporated into the model and time aggregation is explicitly taken into account. Estimation results of the quarterly aggregated expenditures of motor vehicle series and the furnitures/appliances series are shown to be consistent with the implications of the models that incorporate more realistic assumptions such as infrequent purchases and habit persistence preferences. Furthermore, the model estimates of the seasonally unadjusted series of durable expenditures and two subcategories are shown to be consistent with the implications of the stochastic seasonal model. The estimation results are overall consistent with the basic implications of the RE-PIH model of consumption behavior on durable goods. The analysis in this paper suggests that the previous rejection of the RE-PIH model on durable expenditures with quarterly aggregate data may well have been due to model misspecification or to time aggregation bias.

One of the shortcomings of the framework presented in this paper is the assumption that the length of the durable goods purchase interval is identical for all consumers. The simplifying assumption is introduced into the framework to circumvent the complications arising from dealing with the cross-sectional aggregation problem of heterogeneous consumers. A more realistic assumption would be to allow different purchase intervals for different consumers. A possible extension of the present framework is to segregate the consumer population into two or more groups that have different purchase intervals. The approach is similar to the scheme used in the liquidity constrained models where the consumer population is segmented into a liquidity constrained group and unconstrained group. A weighting scheme can be employed to aggregate the segregated groups. A potential problem with such a framework is the arbitrary assignments of weights. One possible method to determine the weights would be to utilize the micro-level survey data on how often consumers actually replace the various durable goods. Theoretically, a fraction could be assigned to each group with different purchase intervals.

Another aspect of the consumption behavior concerning automobiles that we did not pursue in this paper is the growing popularity of leasing automobiles. In NIPA, consumer payments on automobile leasing are included under the services category rather than the consumer durable goods expenditures category. Thus, as the number of consumers who decide to lease their car increase, the composition of the durable goods expenditures will change. That is, the expenditures on motor vehicles will account for less proportion of the total durable expenditures. A possible future work would be to investigate the implication of this recent trend.

Lastly, the observed heteroskedasticity of the durable goods series is not addressed in this paper. The consumer durables series and the two subcategories display an increasing variance across the sample periods. The increasing variance appears to be more pronounced with the seasonally unadjusted series, especially for the furnitures and household appliances series. The increasing seasonal fluctuations may be due to changes in the behavior of economic agents, which suggests that more emphasis should be put on modeling seasonality endogenously rather than using deseasonalized data.

APPENDIX A. BASE MODEL SOLUTION

If a consumer maximizes:

$$E_t \sum_{s=0}^{T-t} (1+\theta)^{-s} U(K_{t+s})$$

subject to

$$\{K_t - (1-\delta)^i K_{t-i} - \sum_{h=0}^{i-1} (1+r)^h Y_{t-h}\} + (1+r)^{-i}\{K_{t+i} - (1-\delta)^i K_t - \sum_{h=0}^{i-1} (1+r)^h Y_{t+i-h}\}$$
$$+ \ldots + (1+r)^{-(T-t)}\{K_T - (1-\delta)^i K_{T-i} - \sum_{h=0}^{i-1} (1+r)^h Y_{T-h}\} = W_t$$

then ;

$$E_t U'(K_{t+i}) = [(1+\theta)/(1+r)]^{i}\, U'(K_t)$$

proof:

At time t, the consumer chooses K_t^* so as to maximize

$$U(K_t) + E_t \sum_{s=1}^{T-t} (1+\theta)^{-s} U(K_{t+s})$$

subject to

$$\{K_t - (1-\delta)^t K_{t-t} - \sum_{h=0}^{t-1}(1+r)^h Y_{t-h}\} + (1+r)^{-t}\{K_{t+t} - (1-\delta)^t K_t - \sum_{h=0}^{i-1}(1+r)^h Y_{t+i-h}\}$$
$$+ \ldots + (1+r)^{-(T-t)}\{K_T - (1-\delta)^i K_{T-t} - \sum_{h=0}^{t-1}(1+r)^h Y_{T-h}\} = W_t$$

Let K_t^* be the optimal consumer durables stock stream.

Consider a variation from this optimum: $K_t = K_t^* + x : K_{t+i} = K_{t+i}^* - (1+r)^i x$, where x is a deviation from the optimum.

Note that the variation also satisfies the budget constraint.

Now we can rewrite the optimization problem as:

Maximize $U(K_t^* + x) + (1+\theta)^{-1} U(K_t^* + x) + (1+\theta)^{-2} U(K_t^* + x) + \ldots$

$+ (1+\theta)^{-(i-1)} U(K_t^* + x) + E_t[(1+\theta)^{-i} U\{K_{t+i}^* - (1+r)^i x\}$

$+ (1+\theta)^{-(i+1)} U\{K_{t+i}^* - (1+r)^i x\} + (1+\theta)^{-(i+2)} U\{K_{t+i}^* - (1+r)^i x\} + \ldots$

$+ (1+\theta)^{-(2i-1)} U\{K_{t+i}^* - (1+r)^i x\} + \ldots]$

The first-order condition with respect to x is

$U'(K_t^* + x)(1 + (1+\theta)^{-1} + (1+\theta)^{-2} + \ldots + (1+\theta)^{-(i-1)})$

$- E_t\, U'\{K_{t+i}^* - (1+r)^i x\}(1+r)^i \{(1+\theta)^{-i} + (1+\theta)^{-(i+1)} + \ldots + (1+\theta)^{-(2i-1)}\} = 0$

In equilibrium, x = 0, thus,

$U'(K_t)(1 + (1+\theta)^{-1} + (1+\theta)^{-2} + \ldots + (1+\theta)^{-(i-1)})$

$= E_t\, U'\{K_{t+i}\}\, (1+r)^i \{(1+\theta)^{-i} + (1+\theta)^{-(i+1)} + \ldots + (1+\theta)^{-(2i-1)}\}$

$\therefore E_t U'(K_{t+i}) = [(1+\theta)/(1+r)]^i\, U'(K_t)$

If utility is quadratic; $U(K_t) = -0.5(K - K_t)^2$, $U'(K_t) = K - K_t$,

then, $K_t = a_0 + a_1K_{t-i} + \mu_t$.

proof:

Substitute $U'(\,) = K - K_t$ into the Euler equation (3.2);

$E_t(K - K_{t+i}) = [(1+\theta) / (1+r)]^i (K - K_t)$

Rearranging the equation gives

$K_{t+i} = a_0 + a_1K_t + \mu_{t+i}$

$K_t = a_0 + a_1K_{t-i} + \mu_t$

APPENDIX B. VARIABLE PURCHASE INTERVAL MODELS

If the expenditure interval is one month; i = 1, then

$\Delta C_1^q = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \beta(\varepsilon_0 + \varepsilon_1 + \varepsilon_2)$

$\Delta C_2^q = \varepsilon_4 + \varepsilon_5 + \varepsilon_6 + \beta(\varepsilon_3 + \varepsilon_4 + \varepsilon_5)$

$\Delta C_3^q = \varepsilon_7 + \varepsilon_8 + \varepsilon_9 + \beta(\varepsilon_6 + \varepsilon_7 + \varepsilon_8)$

The variance and covariances / correlations;

$\mathrm{Var}(\Delta C_t^q) = \{3(1+\beta^2) + 4\beta\}\sigma_\varepsilon^2$

$\mathrm{Cov}(\Delta C_1^q, \Delta C_2^q) = \beta\sigma_\varepsilon^2$

$\mathrm{Cov}(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 3, 4, 5, \ldots$

$\mathrm{Corr}(\Delta C_1^q, \Delta C_2^q) = \beta / \{3(1+\beta^2) + 4\beta\}$

$\mathrm{Corr}(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 3, 4, 5, \ldots$

If i = 2 then

$\Delta C_1^q = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \beta(\varepsilon_{-1} + \varepsilon_0 + \varepsilon_1)$

$\Delta C_2^q = \varepsilon_4 + \varepsilon_5 + \varepsilon_6 + \beta(\varepsilon_2 + \varepsilon_3 + \varepsilon_4)$

$\Delta C_3^q = \varepsilon_7 + \varepsilon_8 + \varepsilon_9 + \beta(\varepsilon_5 + \varepsilon_6 + \varepsilon_7)$

$\mathrm{Var}(\Delta C_t^q) = \{3(1+\beta^2)+2\beta\}\sigma_\varepsilon^2$

$\mathrm{Cov}(\Delta C_1^q, \Delta C_2^q) = 2\beta\sigma_\varepsilon^2$

$\mathrm{Cov}(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 3, 4, 5, \ldots$

$\mathrm{Corr}(\Delta C_1^q, \Delta C_2^q) = 2\beta / \{3(1+\beta^2) + 2\beta\}$

$\text{Corr}(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 3, 4, 5, \ldots$

If i = 3 then

$\Delta C_1^q = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \beta(\varepsilon_{-2} + \varepsilon_{-1} + \varepsilon_0)$

$\Delta C_2^q = \varepsilon_4 + \varepsilon_5 + \varepsilon_6 + \beta(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$

$\Delta C_3^q = \varepsilon_7 + \varepsilon_8 + \varepsilon_9 + \beta(\varepsilon_4 + \varepsilon_5 + \varepsilon_6)$

$\text{Var}(\Delta C_t^q) = 3(1+\beta^2)\sigma_\varepsilon^2$

$\text{Cov}(\Delta C_1^q, \Delta C_2^q) = 3\beta\sigma_\varepsilon^2$

$\text{Cov}(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 3, 4, 5, \ldots$

$\text{Corr}(\Delta C_1^q, \Delta C_2^q) = \beta/(1+\beta^2)$

$\text{Corr}(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 3, 4, 5, \ldots$

If i = 4 then

$\Delta C_1^q = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \beta(\varepsilon_{-3} + \varepsilon_{-2} + \varepsilon_{-1})$

$\Delta C_2^q = \varepsilon_4 + \varepsilon_5 + \varepsilon_6 + \beta(\varepsilon_0 + \varepsilon_1 + \varepsilon_2)$

$\Delta C_3^q = \varepsilon_7 + \varepsilon_8 + \varepsilon_9 + \beta(\varepsilon_3 + \varepsilon_4 + \varepsilon_5)$

$\Delta C_4^q = \varepsilon_{10} + \varepsilon_{11} + \varepsilon_{12} + \beta(\varepsilon_6 + \varepsilon_7 + \varepsilon_8)$

$\text{Var}(\Delta C_t^q) = 3(1+\beta^2)\sigma_\varepsilon^2$

$\text{Cov}(\Delta C_1^q, \Delta C_2^q) = 2\beta\sigma_\varepsilon^2$

$\text{Cov}(\Delta C_1^q, \Delta C_3^q) = \beta\sigma_\varepsilon^2$

$\text{Cov}(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 4, 5, 6, \ldots$

$\text{Corr}(\Delta C_1^q, \Delta C_2^q) = 2\beta/3(1+\beta^2)$

$\text{Corr}(\Delta C_1^q, \Delta C_3^q) = \beta/3(1+\beta^2)$

$Corr(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 4, 5, 6, \ldots$

If i = 5 then

$\Delta C_1^q = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \beta(\varepsilon_{-4} + \varepsilon_{-3} + \varepsilon_{-2})$

$\Delta C_2^q = \varepsilon_4 + \varepsilon_5 + \varepsilon_6 + \beta(\varepsilon_{-1} + \varepsilon_0 + \varepsilon_1)$

$\Delta C_3^q = \varepsilon_7 + \varepsilon_8 + \varepsilon_9 + \beta(\varepsilon_2 + \varepsilon_3 + \varepsilon_4)$

$\Delta C_4^q = \varepsilon_{10} + \varepsilon_{11} + \varepsilon_{12} + \beta(\varepsilon_5 + \varepsilon_6 + \varepsilon_7)$

$Var(\Delta C_t^q) = 3(1+\beta^2)\sigma_\varepsilon^2$

$Cov(\Delta C_1^q, \Delta C_2^q) = \beta\sigma_\varepsilon^2$

$Cov(\Delta C_1^q, \Delta C_3^q) = 2\beta\sigma_\varepsilon^2$

$Cov(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 4, 5, 6, \ldots$

$Corr(\Delta C_1^q, \Delta C_2^q) = \beta / 3(1+\beta^2)$

$Corr(\Delta C_1^q, \Delta C_3^q) = 2\beta / 3(1+\beta^2)$

$Corr(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 4, 5, 6, \ldots$

If i = 6 then

$\Delta C_1^q = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \beta(\varepsilon_{-5} + \varepsilon_{-4} + \varepsilon_{-3})$

$\Delta C_2^q = \varepsilon_4 + \varepsilon_5 + \varepsilon_6 + \beta(\varepsilon_{-2} + \varepsilon_{-1} + \varepsilon_0)$

$\Delta C_3^q = \varepsilon_7 + \varepsilon_8 + \varepsilon_9 + \beta(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$

$\Delta C_4^q = \varepsilon_{10} + \varepsilon_{11} + \varepsilon_{12} + \beta(\varepsilon_4 + \varepsilon_5 + \varepsilon_6)$

$Var(\Delta C_t^q) = 3(1+\beta^2)\sigma_\varepsilon^2$

$Cov(\Delta C_1^q, \Delta C_2^q) = 0$

$Cov(\Delta C_1^q, \Delta C_3^q) = 3\beta\sigma_\varepsilon^2$

$Cov(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 4, 5, 6, \ldots$

$Corr(\Delta C_1^q, \Delta C_2^q) = 0$

$Corr(\Delta C_1^q, \Delta C_3^q) = \beta / (1+\beta^2)$

$Corr(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 4, 5, 6, \ldots$

If $i = 7$ then

$\Delta C_1^q = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \beta(\varepsilon_{-6} + \varepsilon_{-5} + \varepsilon_{-4})$

$\Delta C_2^q = \varepsilon_4 + \varepsilon_5 + \varepsilon_6 + \beta(\varepsilon_{-3} + \varepsilon_{-2} + \varepsilon_{-1})$

$\Delta C_3^q = \varepsilon_7 + \varepsilon_8 + \varepsilon_9 + \beta(\varepsilon_0 + \varepsilon_1 + \varepsilon_2)$

$\Delta C_4^q = \varepsilon_{10} + \varepsilon_{11} + \varepsilon_{12} + \beta(\varepsilon_3 + \varepsilon_4 + \varepsilon_5)$

$Var(\Delta C_t^q) = 3(1+\beta^2)\sigma_\varepsilon^2$

$Cov(\Delta C_1^q, \Delta C_2^q) = 0$

$Cov(\Delta C_1^q, \Delta C_3^q) = 2\beta\sigma_\varepsilon^2$

$Cov(\Delta C_1^q, \Delta C_4^q) = \beta\sigma_\varepsilon^2$

$Cov(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 5, 6, 7, \ldots$

$Corr(\Delta C_1^q, \Delta C_2^q) = 0$

$Corr(\Delta C_1^q, \Delta C_3^q) = 2\beta / 3(1+\beta^2)$

$Corr(\Delta C_1^q, \Delta C_4^q) = \beta / 3(1+\beta^2)$

$Corr(\Delta C_1^q, \Delta C_k^q) = 0$ if $k = 5, 6, 7, \ldots$

If $i = 12$ then

$\Delta C_1^q = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \beta(\varepsilon_{-11} + \varepsilon_{-10} + \varepsilon_{-9})$

$\Delta C_2^q = \varepsilon_4 + \varepsilon_5 + \varepsilon_6 + \beta(\varepsilon_{-8} + \varepsilon_{-7} + \varepsilon_{-6})$

$\Delta C_3^q = \varepsilon_7 + \varepsilon_8 + \varepsilon_9 + \beta(\varepsilon_{-5} + \varepsilon_{-4} + \varepsilon_{-3})$

$\Delta C_4^q = \varepsilon_{10} + \varepsilon_{11} + \varepsilon_{12} + \beta(\varepsilon_{-2} + \varepsilon_{-1} + \varepsilon_0)$

$\Delta C_5^q = \varepsilon_{13} + \varepsilon_{14} + \varepsilon_{15} + \beta(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$

$\Delta C_6^q = \varepsilon_{16} + \varepsilon_{17} + \varepsilon_{18} + \beta(\varepsilon_4 + \varepsilon_5 + \varepsilon_6)$

$\text{Var}(\Delta C_t^q) = 3(1+\beta^2)\sigma_\varepsilon^2$

$\text{Cov}(\Delta C_1^q, \Delta C_5^q) = 3\beta\sigma_\varepsilon^2$

$\text{Cov}(\Delta C_1^q, \Delta C_k^q) = 0$ if $k \neq 5, k \geq 2.$

$\text{Corr}(\Delta C_1^q, \Delta C_5^q) = \beta/(1+\beta^2)$

$\text{Corr}(\Delta C_1^q, \Delta C_k^q) = 0$ if $k \neq 5, k \geq 2.$

If $i = 24$ then

$\Delta C_1^q = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \beta(\varepsilon_{-23} + \varepsilon_{-22} + \varepsilon_{-21})$

$\Delta C_2^q = \varepsilon_4 + \varepsilon_5 + \varepsilon_6 + \beta(\varepsilon_{-20} + \varepsilon_{-19} + \varepsilon_{-18})$

.

.

.

$\Delta C_8^q = \varepsilon_{22} + \varepsilon_{23} + \varepsilon_{24} + \beta(\varepsilon_{-2} + \varepsilon_{-1} + \varepsilon_0)$

$\Delta C_9^q = \varepsilon_{25} + \varepsilon_{26} + \varepsilon_{27} + \beta(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$

$\Delta C_{10}^q = \varepsilon_{28} + \varepsilon_{29} + \varepsilon_{30} + \beta(\varepsilon_4 + \varepsilon_5 + \varepsilon_6)$

$\text{Var}(\Delta C_t^q) = 3(1+\beta^2)\sigma_\varepsilon^2$

$\text{Cov}(\Delta C_1^q, \Delta C_9^q) = 3\beta\sigma_\varepsilon^2$

$\text{Cov}(\Delta C_1^q, \Delta C_k^q) = 0$ if $k \neq 9, k \geq 2.$

$\text{Corr}(\Delta C_1^q, \Delta C_9^q) = \beta/(1+\beta^2)$

$\text{Corr}(\Delta C_1^q, \Delta C_k^q) = 0$ if $k \neq 9, k \geq 2.$

The above result can be summarized as follows;

If $i = 1$; then $Corr(\Delta C_1^q, \Delta C_2^q) = \beta / \{3(1+\beta^2) + 4\beta\}$ ... MA (1)

If $i = 2$; then $Corr(\Delta C_1^q, \Delta C_2^q) = 2\beta / \{3(1+\beta^2) + 2\beta\}$ ... MA (1)

where $\beta = -(1-\delta)^i$

If $i = 3j$; $j=1,2,\ldots$, then $Corr(\Delta C_1^q, \Delta C_{1+j}^q) = \beta / (1+\beta^2)$ ... MA (|| j ||)

where $\beta = -(1-\delta)^{3j}$

If $i = 3j + k$; ($j=1,2,\ldots$; $k=1,2,\ldots$)

then $Corr(\Delta C_1^q, \Delta C_{1+j}^q) = (3-k)\beta / 3(1+\beta^2)$

$Corr(\Delta C_1^q, \Delta C_{2+j}^q) = k\beta / 3(1+\beta^2)$ ... MA (|| j, j+1 ||)

where $\beta = -(1-\delta)^{3j+k}$

APPENDIX C. HABIT PERSISITENCE MODEL SOLUTION

If a consumer maximizes:

$$E_t \sum_{s=0}^{T-t} (1+\theta)^{-s} U(K_{t+s} - \phi K_{t+s-i})$$

subject to

$$\{K_t - (1-\delta)^i K_{t-i} - \sum_{h=0}^{i-1} (1+r)^h Y_{t-h}\} + (1+r)^{-i} \{K_{t+i} - (1-\delta)^i K_t - \sum_{h=0}^{i-1} (1+r)^h Y_{t+i-h}\}$$
$$+ \ldots + (1+r)^{-(T-t)} \{K_T - (1-\delta)^i K_{T-i} - \sum_{h=0}^{i-1} (1+r)^h Y_{T-h}\} = W_t$$

then ;

$$E_t U'(K_{t+i} - \phi K_t) = [(1+\theta)^i / \{(1+r)^i + \phi\}] \, U'(K_t - \phi K_{t-i})$$

proof:

At time t, the consumer chooses K_t^* so as to maximize

$$U(K_t - \phi K_{t-i}) + E_t \sum_{s=1}^{T-t} (1+\theta)^{-s} U(K_{t+s} - \phi K_{t+s-i})$$

subject to

$$\{K_t-(1-\delta)^i K_{t-i}-\sum_{h=0}^{i-1}(1+r)^h Y_{t-h}\}+(1+r)^{-i}\{K_{t+i}-(1-\delta)^i K_t-\sum_{h=0}^{i-1}(1+r)^h Y_{t+i-h}\}$$
$$+\ldots+(1+r)^{-(T-t)}\{K_T-(1-\delta)^i K_{T-i}-\sum_{h=0}^{i-1}(1+r)^h Y_{T-h}\}=W_t$$

Let K_t^* be the optimal consumer durables stock stream.

Consider a variation from this optimum: $K_t = K_t^* + x$: $K_{t+i} = K_{t+i}^* - (1+r)^i x$, where x is a deviation from the optimum.

Note that the variation also satisfies the budget constraint.

Now we can rewrite the optimization problem as:

Maximize $U(K_t^* + x - \phi K_{t-i}) + (1+\theta)^{-1}U(K_t^* + x - \phi K_{t-i}) + (1+\theta)^{-2}U(K_t^* + x - \phi K_{t-i}) + \ldots$

$+ (1+\theta)^{-(i-1)}U(K_t^* + x - \phi K_{t-i}) + E_t[(1+\theta)^{-i}U\{K_{t+i}^* - (1+r)^i x - \phi(K_t^* + x)\}$

$+ (1+\theta)^{-(i+1)}U\{K_{t+i}^* - (1+r)^i x - \phi(K_t^* + x)\} + (1+\theta)^{-(i+2)}U\{K_{t+i}^* - (1+r)^i x - \phi(K_t^* + x)\}$

$+ \ldots + (1+\theta)^{-(2i-1)}U\{K_{t+i}^* - (1+r)^i x - \phi(K_t^* + x)\} + \ldots]$

The first-order condition with respect to x is

$$U'(K_t^* + x - \phi K_{t-i})(1 + (1+\theta)^{-1} + (1+\theta)^{-2} + \ldots + (1+\theta)^{-(i-1)})$$
$$- E_t U'\{K_{t+i}^* - (1+r)^i x - \phi(K_t^* + x)\}\{(1+r)^i + \phi\}\{(1+\theta)^{-i} + (1+\theta)^{-(i+1)} + \ldots$$
$$+ (1+\theta)^{-(2i-1)}\} = 0$$

In equilibrium, x = 0, thus,

$$U'(K_t - \phi K_{t-i})(1 + (1+\theta)^{-1} + (1+\theta)^{-2} + \ldots + (1+\theta)^{-(i-1)})$$
$$= E_t U'\{K_{t+i} - \phi K_t\}\{(1+r)^i + \phi\}\{(1+\theta)^{-i} + (1+\theta)^{-(i+1)} + \ldots + (1+\theta)^{-(2i-1)}\}$$
$$\therefore E_t U'(K_{t+i} - \phi K_t) = [(1+\theta)^i / \{(1+r)^i + \phi\}]\, U'(K_t - \phi K_{t-i})$$

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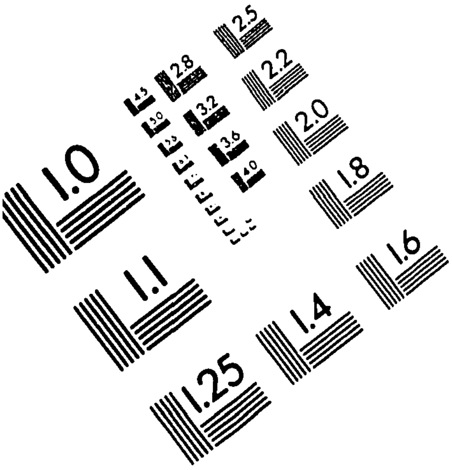
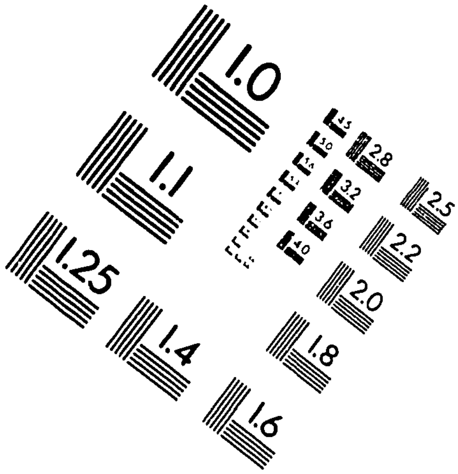
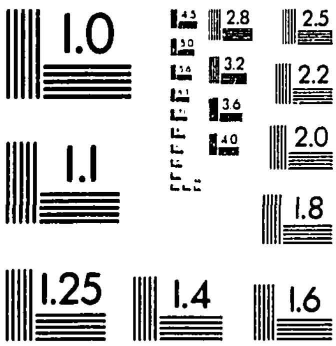
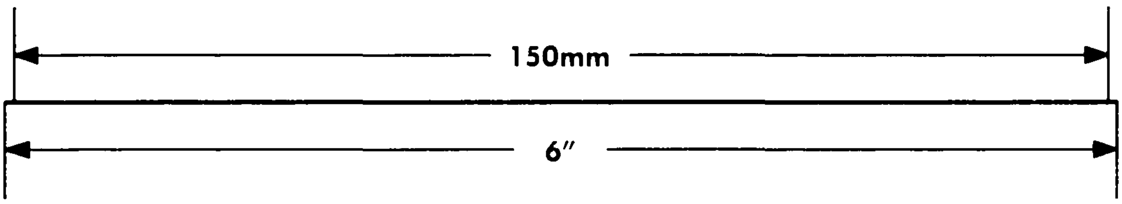
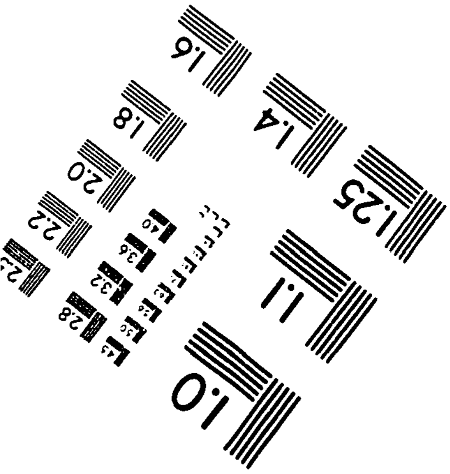
IMAGE EVALUATION TEST TARGET (QA-3)

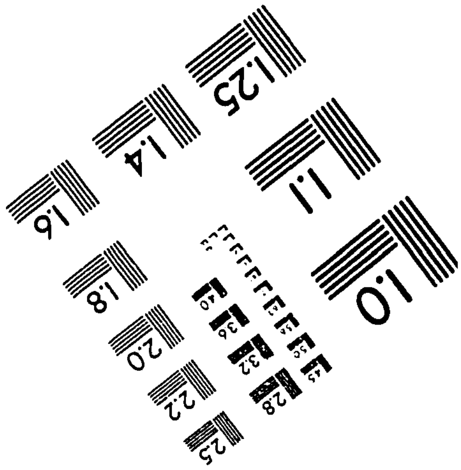












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